Combining switching factors and filtering operators in GSA to analyze models with climatic inputs

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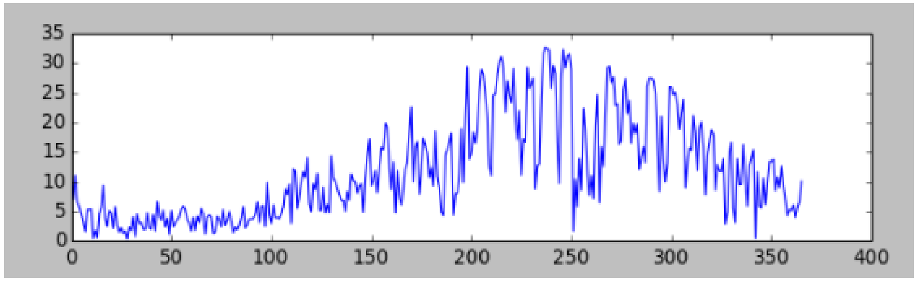
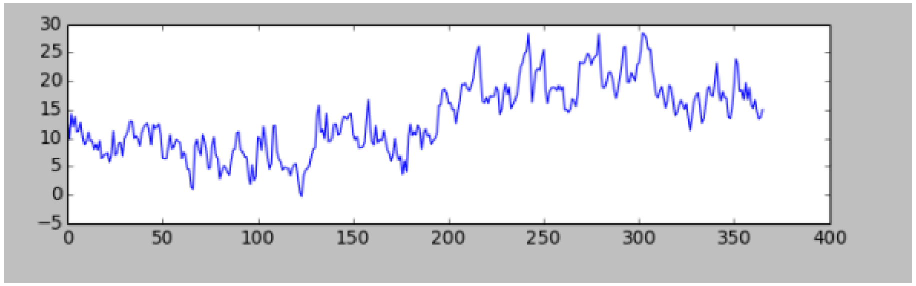
Typical structure of model inputs in crop models

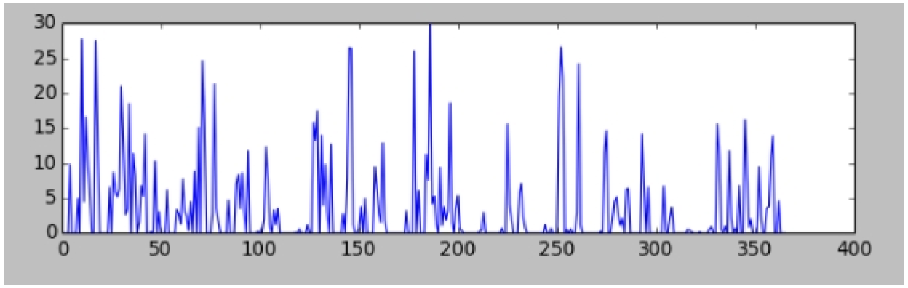
$$X(t) = [x_{rad}(t), x_{temp}(t), x_{rain}(t), x_{et_0}(t)]$$
$$y = f(X(t), p)$$
$$y \in \mathbb{R}$$



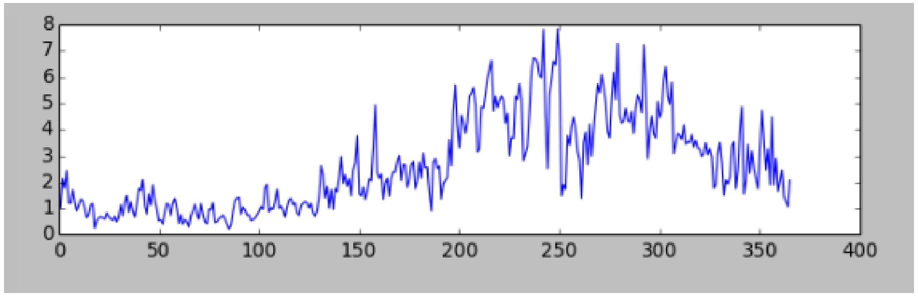
x_rad(t)



x_temp(t)



x_rain(t)



x_et0(t)

Objective: Simplifying the structure of the model inputs

- The method applies on the vector of functional inputs

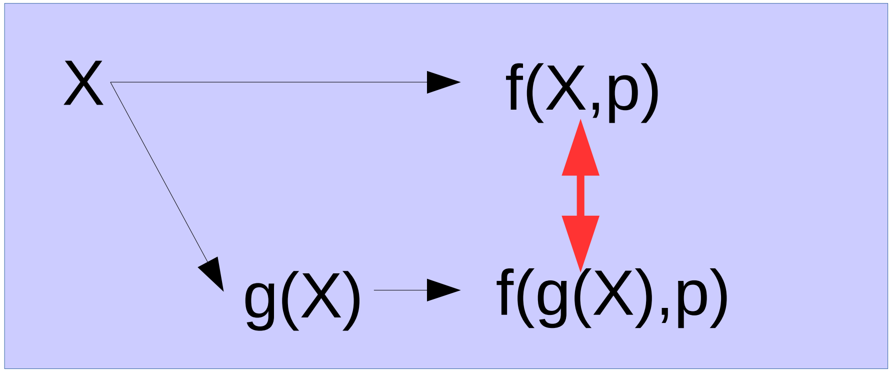
$$X(t)=[x_{rad}(t), x_{temp}(t), x_{rain}(t), x_{et_0}(t)]$$

- **Principle**:

  Given a simplification operator g, test if:

$$f(X, p) \simeq f(g(X), p)$$

Expected benefits for environmental models



Model understanding:

g(X) can highlight sensitive features of X

=> impact of climate on model outputs:
critical periods, climatic events

Model simplification:

g(X) can be easier to acquire or parametrize

=> facilitate acquisition of model inputs (temporal resolution)

=> guiding a re-parametrization of model inputs

Combining switching factors and filtering operators

Switching factors *(Crosetto and Tarantola, 2001)*

- **Principle**
  - Addition of a Bernoulli distributed factor to switch between 2 model versions (one with noisy inputs)
- **Properties**
  - **Sobol indices** associated to the switch are not the Sobol indices of the functional input *(Iooss and Ribatet, 2009)*
  - **But** they are a measure of the sensitivity of the output to a perturbation of the initial model

In the following **switching factor = η**

Application in the context of input simplification

$$f_g(\eta, X, p) = \begin{cases} f(X, p) & \text{if } \eta = 0 \\ f(g(X), p) & \text{if } \eta = 1 \end{cases}$$

- **With labeling variables** *(Liburne and Tarantola, 2009)*

  l = label of climatic year

$$\tilde{f}_g(\eta, l, p) = \begin{cases} f(X_l, p) & \text{if } \eta = 0 \\ f(g(X_l), p) & \text{if } \eta = 1 \end{cases}$$

- **With several independent switching variables**

$$\tilde{f}_{g_{1,..}, g_q}\left(\begin{matrix} \eta_1 \\ .. \\ \eta_q \end{matrix}, l, p\right) = f\left(\begin{matrix} \eta_1 . g_1(X_l^1) + (1-\eta_1) . X_l^1 \\ .. \\ \eta_q . g_q(X_l^q) + (1-\eta_q) . X_l^q \end{matrix}, p\right)$$

Combining switching factors and filtering operators

$$f(X,p) \overset{?}{\simeq} f(g(X),p)$$

- $f_g(\eta, X, p) = \begin{cases} f(X,p) & \text{if } \eta = 0 \\ f(g(X),p) & \text{if } \eta = 1 \end{cases}$

- **Total Sensitivity Index (TSI) of switching factors**

  If TSI(η) is negligible, then η can be fixed, and X can be replaced by g(X)

Link between TSI of switching factors and error criteria

- **TSI are used to extend an error measure to a global exploration**

Fixed (X,p)	Varying (X), $X_1,..,X_N$	Varying (X,p), $X_1,..,X_{N,}$
Error estimation : ε $\epsilon = \| f(X,p) - f(g(X),p) \|$	***Error estimation approach: MSE*** $y_\eta^l = f(\eta, l)$ $\quad MSE = \frac{1}{N}\sum_{l=1}^{N}(y_0^l - y_1^l)^2$	
	GSA-based : TSI(η) $\tilde{f}(l,\eta) = f(\eta \cdot g(X_l)) + (1-\eta) \cdot f(X_l)$ $l \sim DU(N), \eta \sim B(1,0.5)$	***GSA-based : TSI(η)*** $\tilde{f}(l,\eta,p) = f(\eta \cdot g(X_l) + (1-\eta) \cdot X_l, p)$ $l \sim DU(N), \eta \sim B(1,0.5), p \sim L_p$

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- **In simplified contexts (only X varies), TSI and mean square error are strongly related**

$$y_\eta^l=f(\eta,l)$$

$$TSI_\eta=\frac{1}{NV}\sum_{l=1}^{N}\left(\frac{y_0^l-y_1^l}{2}\right)^2=\frac{1}{4V}MSE$$

The ToyCrop model

- Climatic inputs: $X(t) = \left[x^t_{rad}, x^t_{tmoy}, x^t_{et0}, x^t_{rain}\right]$
- Other inputs: $p = [t_1, t_2, \tau_{tmoy}, \tau_{FTSW}, k_c, TTSW, CN, b_0, ATSW_0]$

$$y = \sum_{t_1}^{t_2} \Delta b^t$$

$$\Delta b_t = RUE^t \cdot x^t_{rad}$$

$$RUE^t = (x^t_{tmoy} < \tau_{tmoy}) \cdot \min(1, \frac{FTSW^{t-1}}{\tau_{FTSW}})$$

- Output=biomass
- Radiation driven growth
- Two limiting factors
  temperature + water stress

$$ATSW^t_{tmp} = ATSW^{t-1} + x^t_{rain} - Q\left(CN, x^{t-5,.,t}_{rain}\right) - k_c \cdot x^t_{et0} \cdot \min\left(1, \frac{FTSW^{t-1}}{\tau_{FTSW}}\right)$$

$$ATSW^t = \min\left(TTSW, \max(0, ATSW^t_{tmp})\right)$$

$$FTSW^t = \frac{ATSW^t}{TTSW}$$

Water stress defined from a simple water balance

⇒ Realistic (processes, structure)

⇒ Parsimonious : qualitative validation

Experiment 1: Can some climate variables be replaced by their inter-annual mean g1?

Inter-annual averaging filter over N climatic years

$$g_1(X(t))=\frac{1}{N}\sum_{1}^{N} X_i(t)$$

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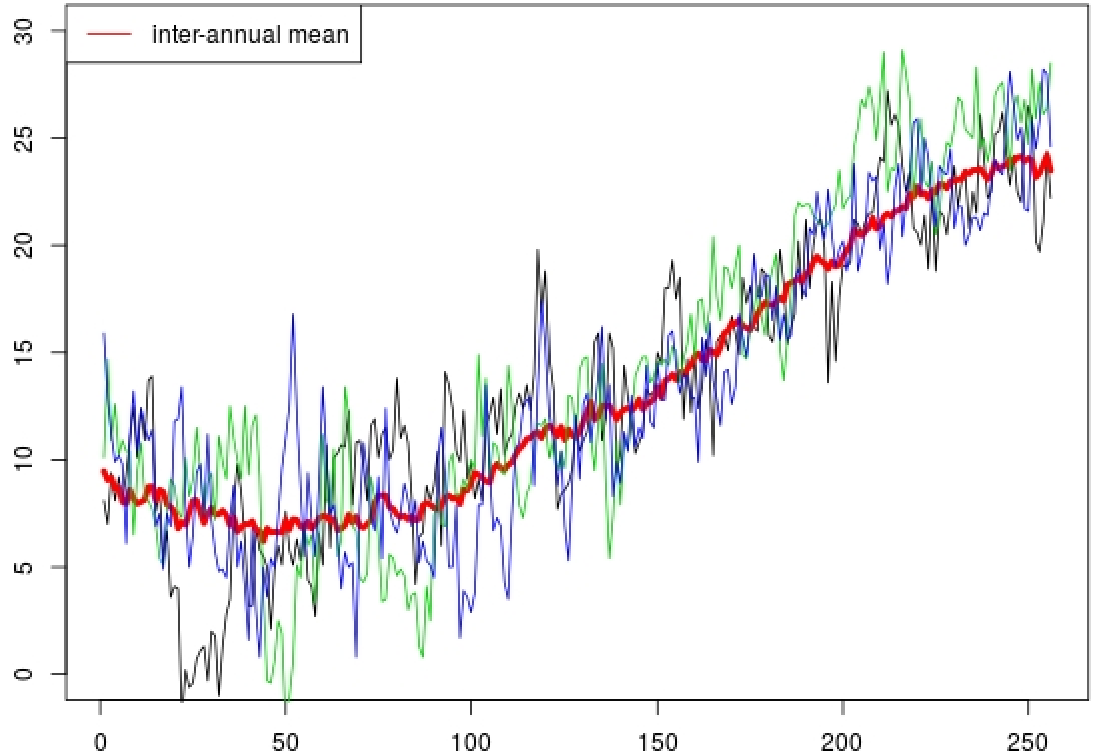


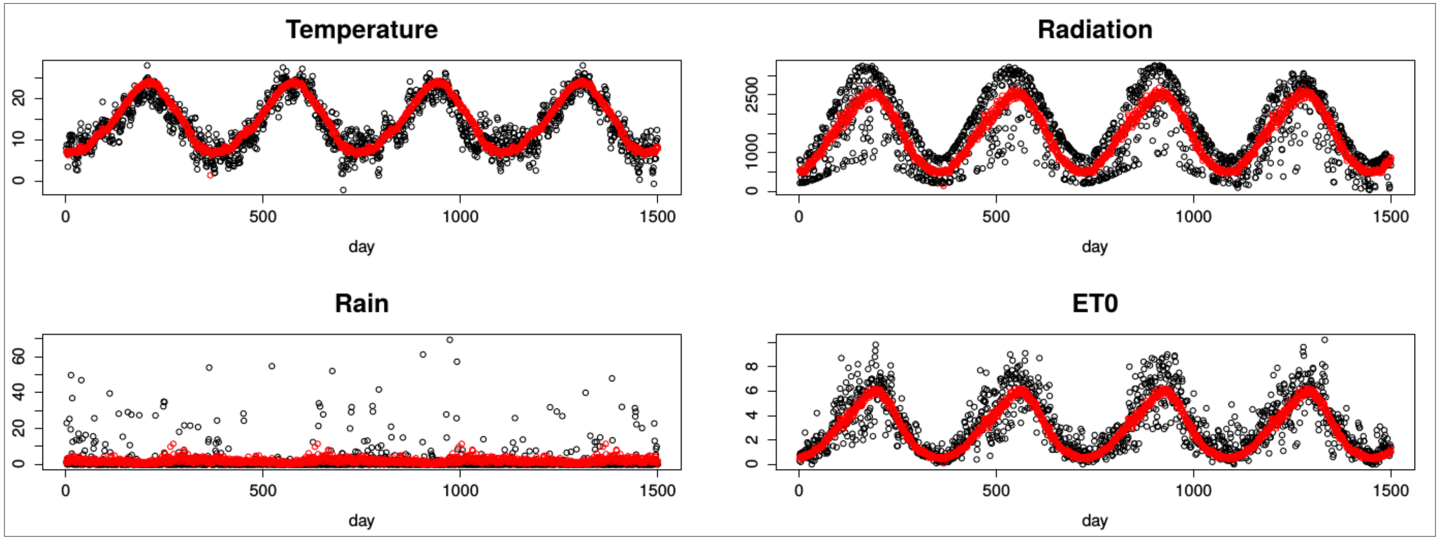
day

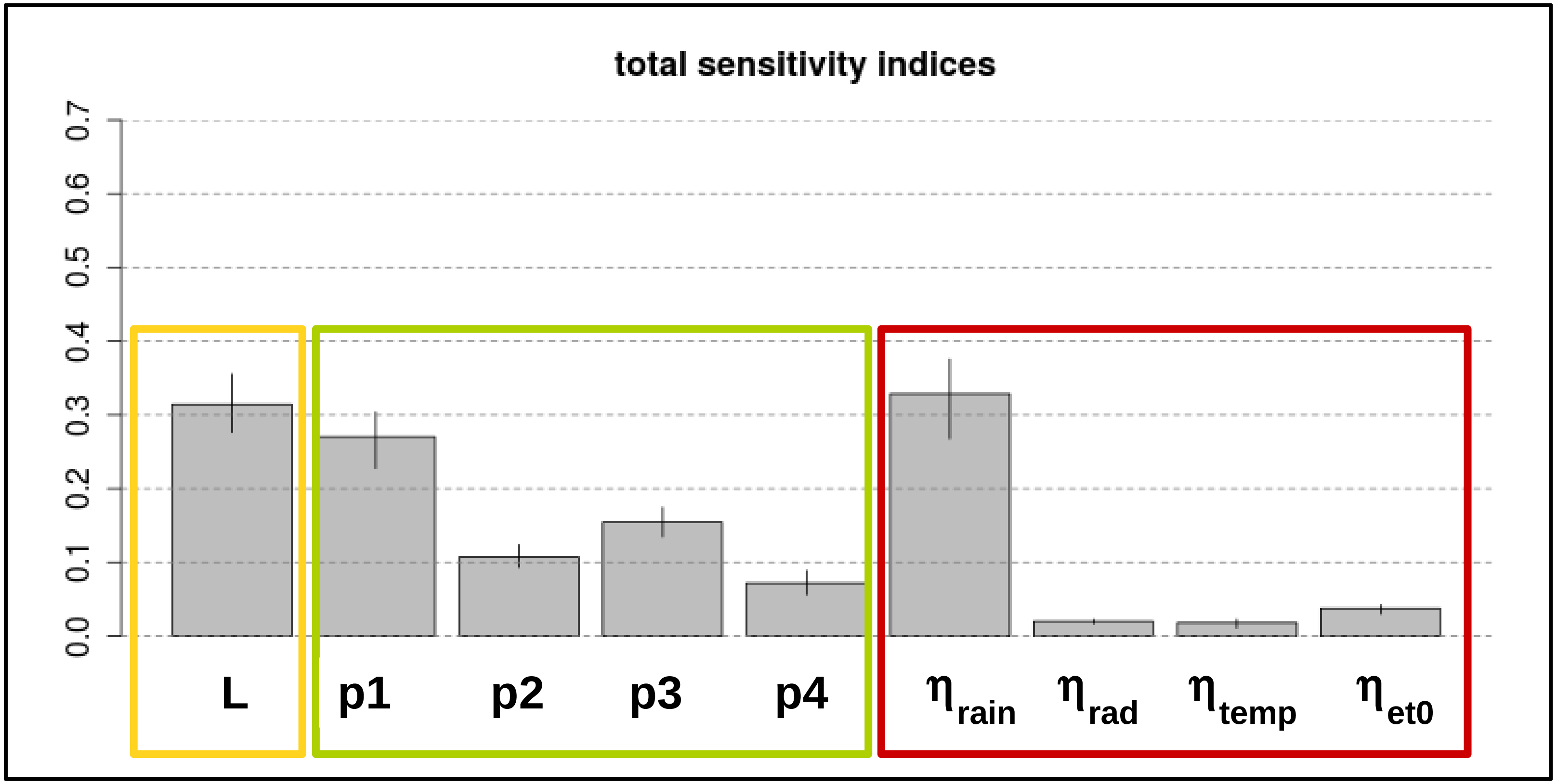
Experiment 1: Can some climate variables be replaced by their inter-annual mean g1?

FACTOR NAME	DEFINITION	DISTRIBUTION
p1	Temperature threshold	U[20,30]
p2	Maximum Soil water Content	U[150,250]
p3	Transpiration rate (cultural coefficient)	U[0.5,0.8]
p4	Runoff strength	B(1,0.5)
L	Label for climatic year	DU(42)
η_{rain}	Switching factor for rain (g1)	B(1,0.5)
η_{rad}	Switching factor for radiation (g1)	B(1,0.5)
η_{temp}	Switching factor for temperature (g1)	B(1,0.5)
η_{et0}	Switching factor for evapotranspiration (g1)	B(1,0.5)

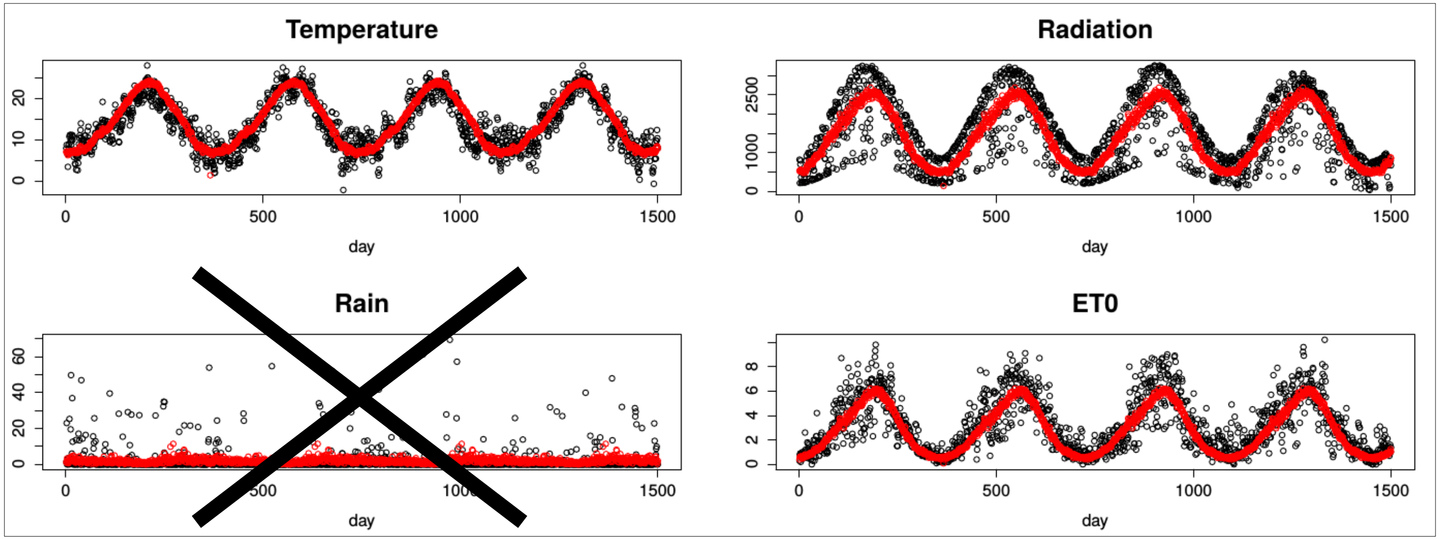
with $$g_1(X(t)) = \frac{1}{N}\sum_{1}^{N} X_i(t)$$

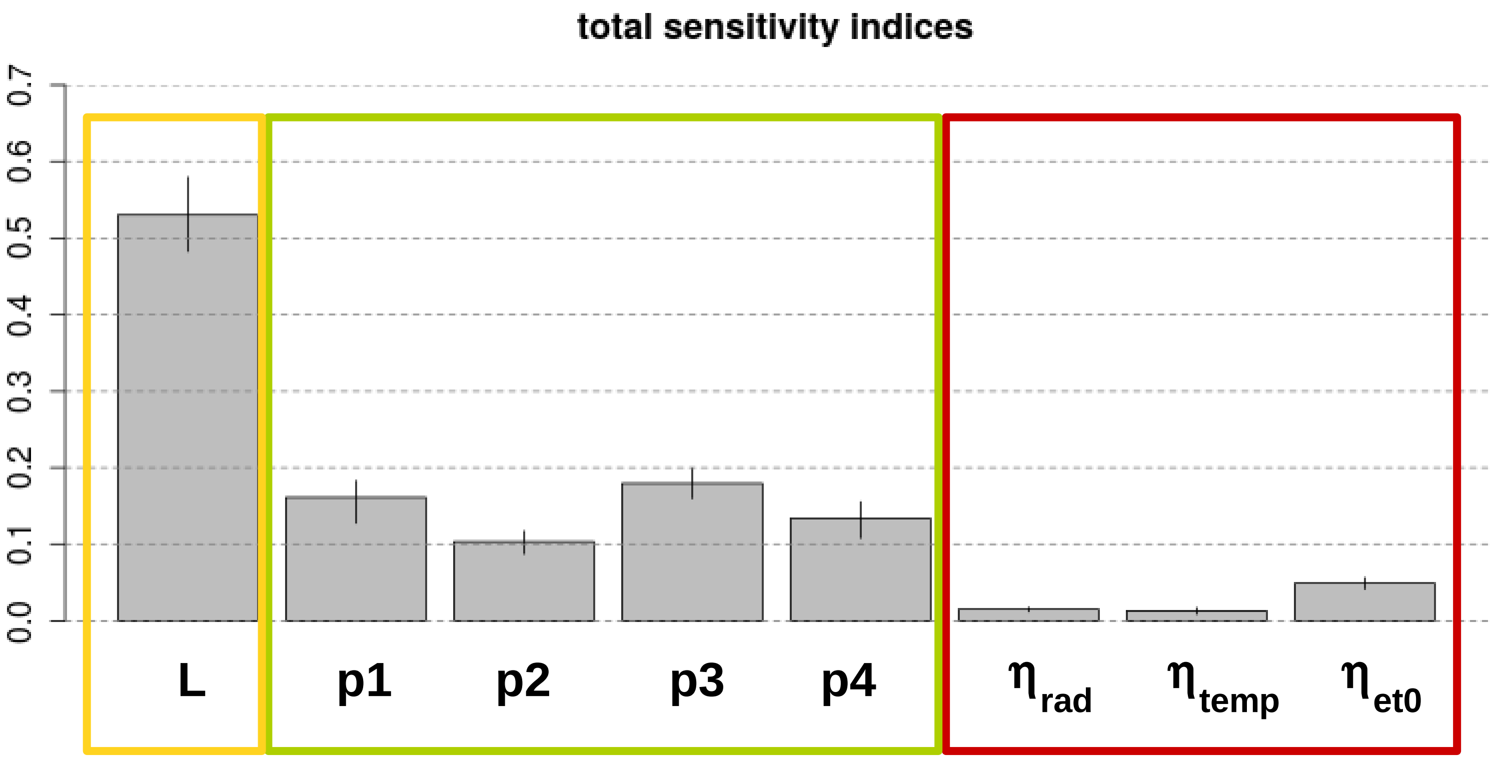
Results with switch on the 4 climatic variables





Results with switch on 3 climatic variables

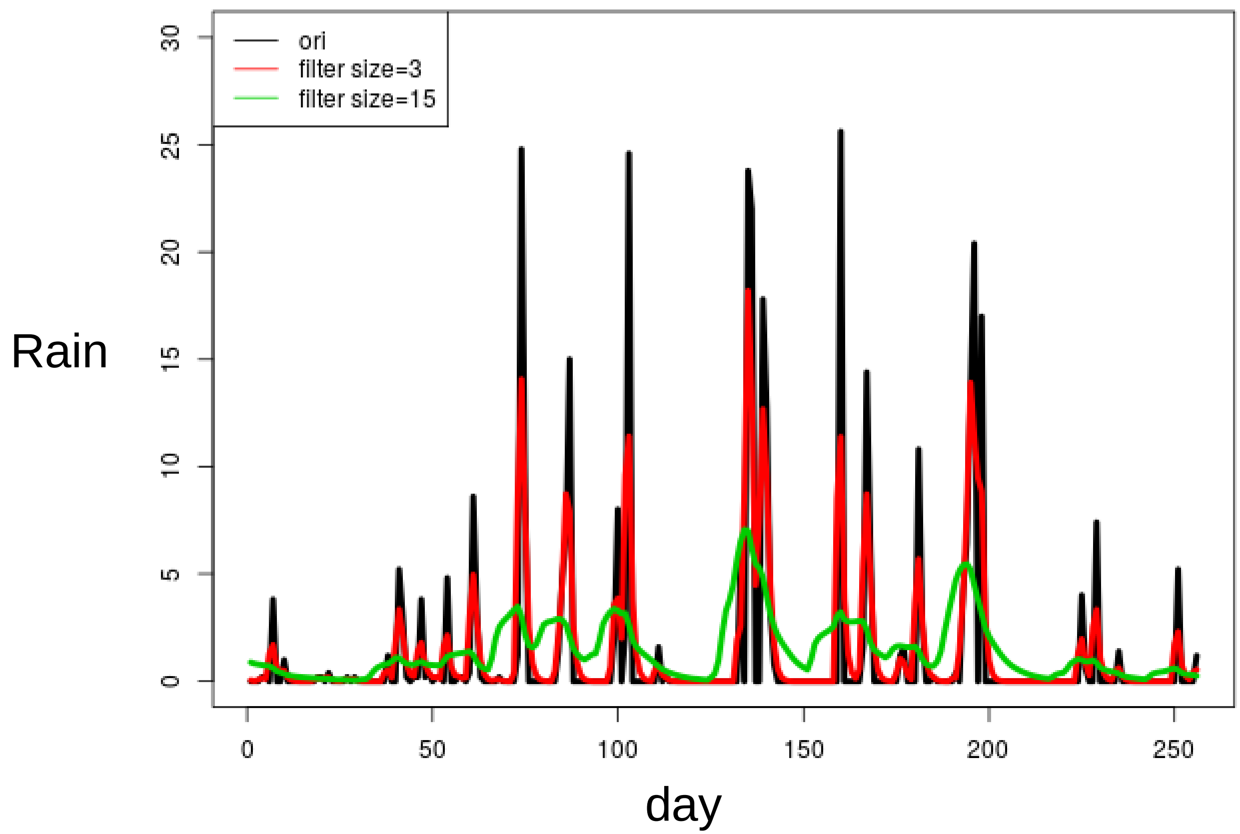




Experiment 2: Can we gain insight into the impact of rain?

- **Parameterized low pass filter**

$$g_2(X,\sigma)(t)=\int X(t-u)G_\sigma(u)\,du$$



Filter size= 3
Filter size =15

Experiment 2: Can we gain insight into the impact of rain? Can we lower the temporal resolution of the rain input?

FACTOR NAME	DEFINITION	DISTRIBUTION
p1	Temperature threshold	U[20,30]
p2	Maximum Soil water Content	U[150,250]
p3	Transpiration rate (cultural coefficient)	U[0.5,0.8]
L	Label for climatic year	DU(42)
η_{rain}	**Switching factor for rain (g2(σ))**	B(1,0.5)
η_{rad}	Switching factor for radiation (g1)	B(1,0.5)
η_{temp}	Switching factor for temperature (g1)	B(1,0.5)
η_{et0}	Switching factor for evapotranspiration (g1)	B(1,0.5)

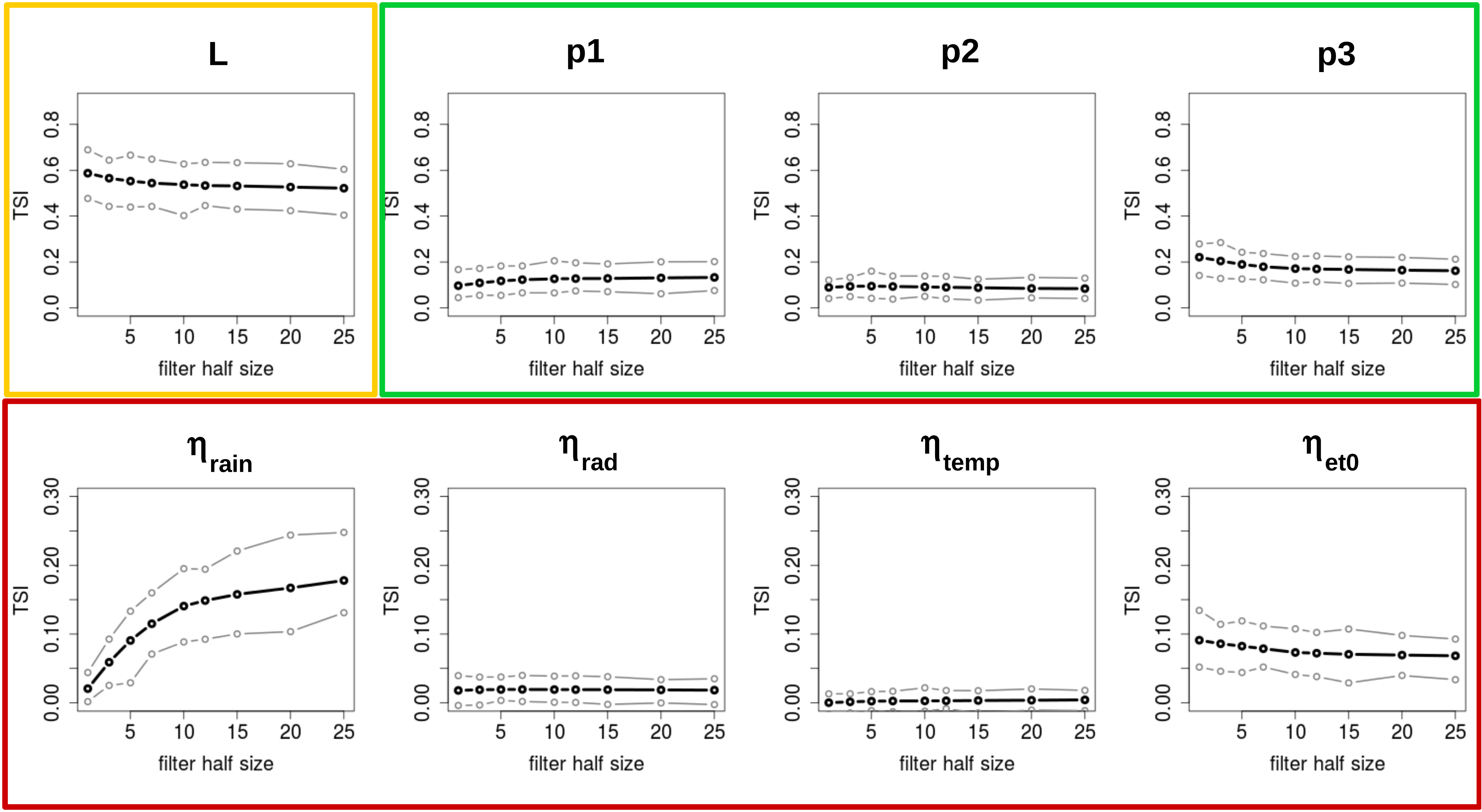
With $g_2(X,\sigma)(t)=\int X(t-u)G_\sigma(u)\,du$

2 tests : **low and high runoff setting**

Repetition of the experiment for different filter widths : TSI(σ)

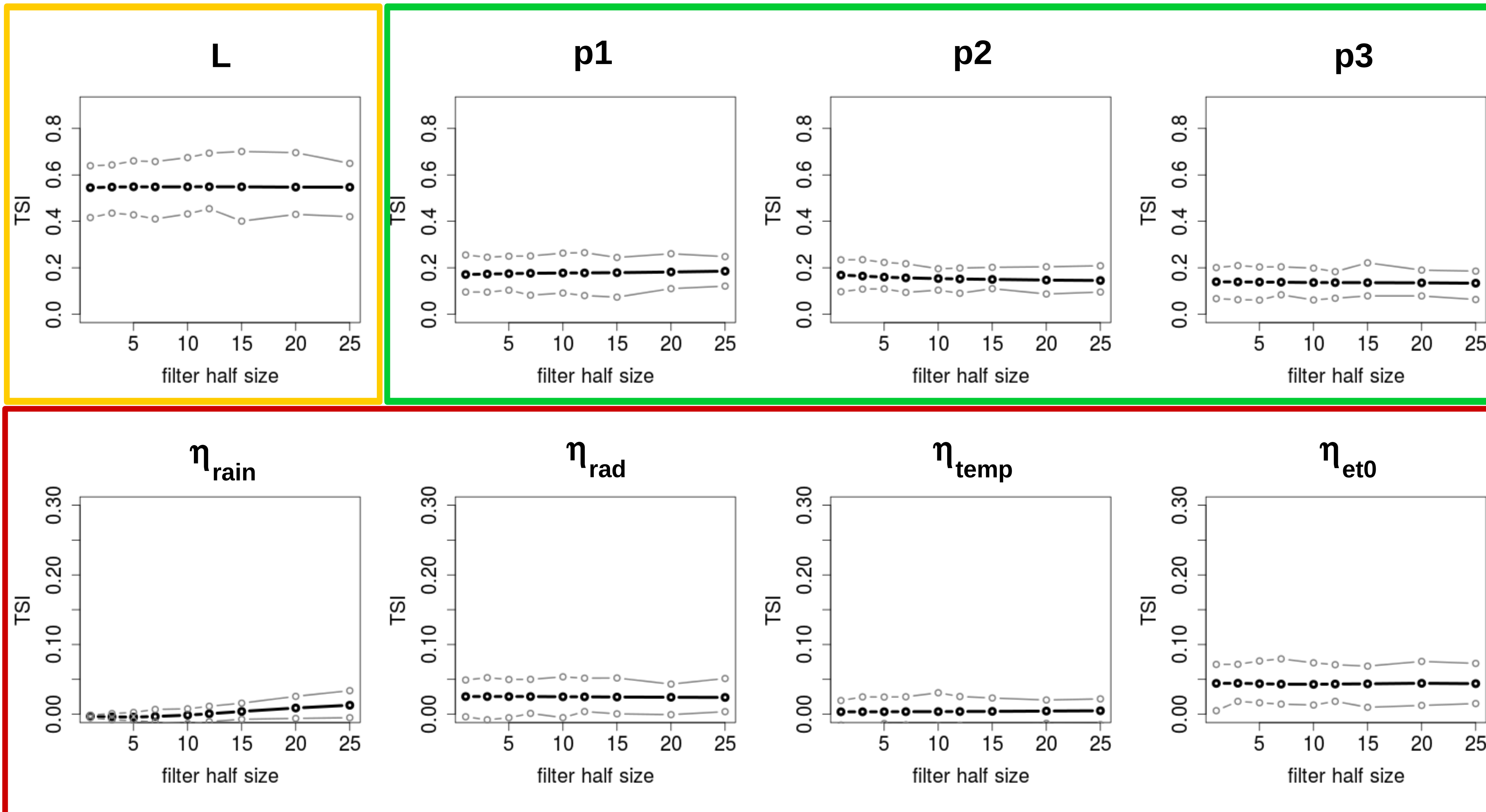
Result: Sensitivity indices when varying the size of temporal averaging

High run-off setting



Result: sensitivity indices when varying the size of temporal averaging

Low run-off setting



Conclusions

- We have applied a switching factor approach to analyze models with functional climatic inputs
- Switching factors are combined with a priori simplification operators that are tested through GSA
- TSI of the switching variables are used as an sound extension of error criteria

Conclusions and Perspectives

- Depending on the choice of the filter, the same approach can be used to study different questions
- Other filters can be considered :
  - edge preserving filter => impact of preserving high values
  - MR filter => localization of sensitive details in terms of position and resolution
- Next step will involve complex crop models (STICS crop model)

Conclusions and Perspectives

- Depending on the choice of the filter, the same approach can be used to study different questions
- Other filters can be considered :
  - edge preserving filter => impact of preserving high values
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- Next step will involve complex crop models (STICS crop model)

Thank you for your attention!