Regionalized sensitivity analysis with respect to multiple outputs - and an application for real-time building space exploration

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Background

Building design involves a large number of design parameters and performance indicators. The Monte Carlo method enables the modeler to perform thousands of building performance simulations representing a global design space. To explore such multivariate data (Factor Mapping [1]), the parallel coordinate plot (PCP) is a popular tool, because it is easy to use in "real-time" – even for multiple decision-makers.

However, the PCP becomes unmanageable if it contains many variables, e.g. more than 10–15. Since building simulations typically involve a lot more parameters, we would like to reduce the number of variable inputs (Factor Fixing [1]) while considering their influence towards multiple outputs. Moreover, we would like a method to highlight changes in the PCP, which would allow us to use more variables in the PCP.

Ideas

The ideas are to apply the Kolmogorov-Smirnov two-sample statistics (KS2) to:

1) rank inputs with respect to multiple outputs (denoted TOM)
2) highlight changes in the PCP in real-time (denoted TOR)

Building case study

To test the proposed sensitivity measures, TOM and TOR, we consider the design of a 15.000 m² educational institution. The "variability" of 10 design parameters are described by uniform distributions (Table 1). Quasi-random sampling (Sobol's LP_τ) is used to sample 5.000 simulations. The simulation software consists of a normative model (ISO 13790) to assess energy demand and "overtemperature". In addition, a regression model is used to assess daylight factor in lecture rooms.

Table 1. Distributions for 10 design parameters

Input parameters	Unit	Uniform min – max	Discrete
Window-facade-ratio	%	40 – 80	
Solar panels	*m²*		0; 100; 200
Reflectance, room mean	-	0.4 – 0.6	
Solar Heat Gain Coef.	-		0.25; 0.32; 0.41;0.5
Side fins (louvres)	°	0 – 45	
U-value, windows	*W/m² K*		0.75; 0.8; 0.85; 0.9
Heat capacity, building	*Wh/m²*		60; 70; 80; 90; 100
Venting	*l/s m²*	0.9 – 1.8	
U-value, facade	*W/m² K*	0.12 – 0.20	
Infiltration	*l/s m²*		0.06; 0.07; 1.0

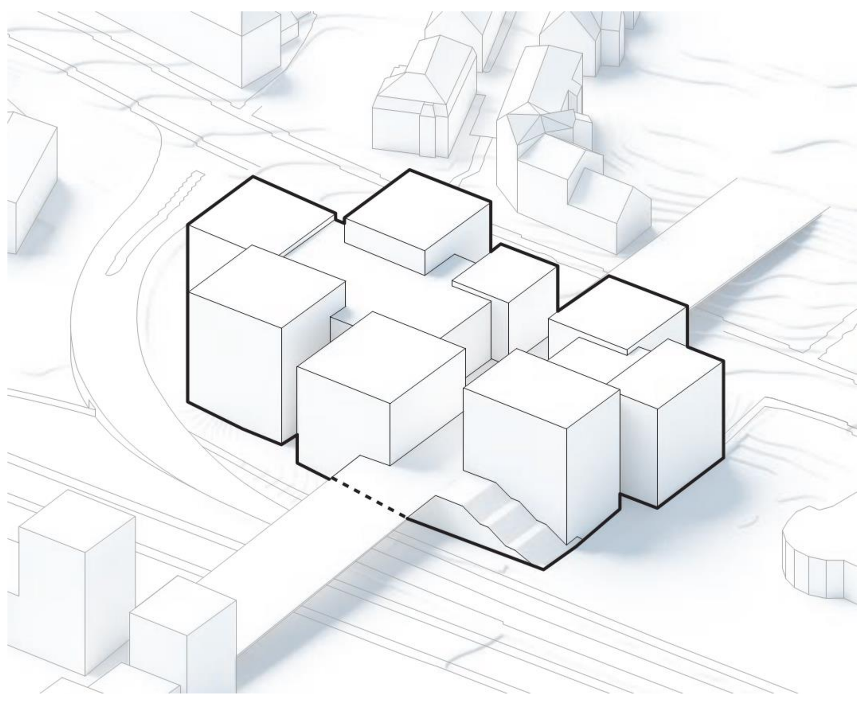


Figure 1. Case building (EFFEKT architects)

Sensitivity towards multiple outputs (TOM)

Here, we present a method to assess inputs' sensitivity towards multiple outputs [2]. The method relies on Monte Carlo Filtering and therefore belongs to the category of Regionalized Sensitivity Analysis. The idea is to apply random filters to all outputs in order to split a set of simulations, S_A, into a "behavioural" subset, S_B, and "non-behavioural" subset, S_N. Using Kolmogorov-Smirnov two-sample statistics, we then calculate the maximum distances, D_i, between the cumulative distributions of S_N and S_A for each input i. These steps are repeated J number of times, i.e. until we reach convergence of the average values of D_i's (inspired by [3]).

Figure 2 illustrates how we randomly split 10 simulations J times. First, we assign an index to each simulation. Then, we sort each output while keeping a reference to the simulations' indices. For each output, we obtain a random subset, S_{yj}, by taking a random starting point and selecting Q simulations above this value. The intersecting indices from these subsets, S_{yj}, constitute the j^{th} behavioural subset, $S_{B,j}$. After J repetitions, we calculate the average D_i's for all inputs. From these, we define a relative sensitivity measure, SA_{TOR}, as in equation (1), which is used to rank inputs according to multiple outputs.

(1) $$SA_{TOR,i} = \frac{D_{i,av}}{\sum_i D_{i,av}}$$

(2) $$Q = N \cdot 0.5^{1/m}$$ N = number of simulations m = number of outputs

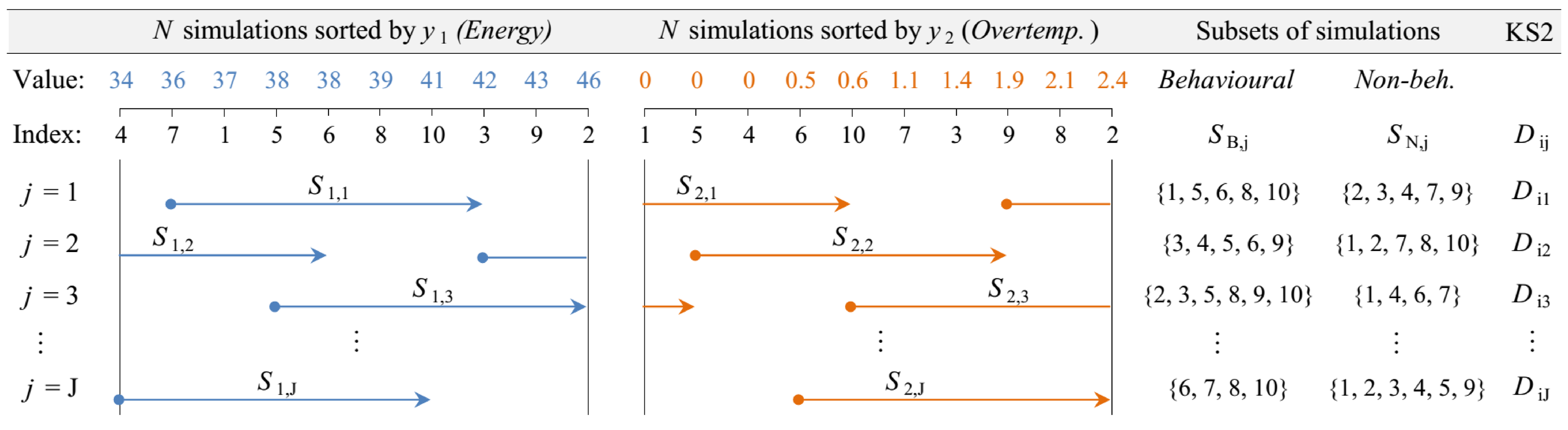


Figure 2. Applying random filters to split simulations into subsets, S_B and S_N, J times (from [2])

We set the size of the random subsets according to equation (2). This causes the behavioural set and non-behavioural sets to be roughly of the same size, which results in more accurate D_i measures. For our case study, the average D_i's converges after ~100 repetitions. In addition, the *average* D_i values provides more stable ranking when compared to the *median* and *maximum* values of D_i. Finally, the ranking depends on the number of simulations available, N. Here, the ranking converged after ~1.000 simulations [2].

Table 1 shows how TOM (J=200, N=5.000) compares with Standardized Regression Coefficients (SRC, with R^2-values of 0.96, 0.42, and 0.96), and Morris Elementary Effects (EE, with 450 trajectories). For the two rightmost columns, we duplicated the daylight output 5 times to see what happens if outputs are highly correlated. For comparison, we simply summed the SRC percentages for each output, and calculated their relative sensitivity. We notice that TOM is less influenced by Daylight than compared to 'Multi-SRC', and thus gives less weighting to correlated outputs.

Table 1. Sensitivity measures when considering outputs separately or at the same time

Parameter	E, O, D		Energy Demand (E)						Overtemperature (O)						Daylight Factor (D)						E, O, 5 x D			
	TOM		SRC		EE		TOM		SRC		EE		TOM		SRC		EE		TOM		TOM		'Multi-SRC'	
Win-fac-ratio	1	23%	2	27%	2	28%	2	24%	2	31%	2	33%	2	27%	1	34%	1	35%	1	34%	1	26%	1	33%
Solar panels	2	19%	1	31%	1	29%	1	36%	-	0%	-	0%	10	1%	-	0%	-	0%	10	1%	3	11%	5	4%
Reflectance	3	17%	9	2%	9	2%	8	3%	7	1%	-	0%	5	5%	2	34%	2	35%	2	32%	2	22%	2	25%
SHGC	4	13%	7	4%	4	8%	5	6%	1	40%	1	42%	1	40%	4	13%	4	11%	4	9%	5	10%	3	15%
Side fins	5	9%	10	1%	10	2%	10	2%	5	4%	5	4%	6	4%	3	18%	3	19%	3	15%	4	11%	4	14%
U-value win.	6	5%	3	13%	3	11%	3	9%	6	3%	6	2%	7	4%	-	0%	-	0%	9	1%	7	4%	7	2%
Heat capacity	7	5%	4	9%	5	8%	4	6%	4	8%	4	7%	4	6%	-	0%	-	0%	5	3%	8	4%	6	3%
Venting	8	4%	8	3%	8	3%	9	3%	3	12%	3	12%	3	10%	-	0%	-	0%	6	3%	6	5%	8	2%
U-value fac.	9	3%	6	5%	7	4%	7	4%	-	0%	7	1%	8	1%	-	0%	-	0%	7	1%	9	4%	10	1%
Infiltration	10	3%	5	6%	6	5%	6	5%	-	0%	-	0%	9	1%	-	0%	-	0%	8	1%	10	3%	9	1%

Real-time highlight of changes in the PCP (TOR)

With the TOR approach, we suggest using KS2 to highlight the coordinates that changes the most when users apply filters in the parallel coordinate plot [2]. The user-defined filters splits the entire set of simulations, S_A, into a behavioural set S_B and non-behavioural set S_N. Each time a filter is applied, we calculate and compare the relative sizes of the maximum distances D_i between the cumulative distributions of the S_B and S_A for every (non-filtered) parameter. The results are illustrated with bar plots just below the PCP's on Figure 3. It works with both inputs and outputs.

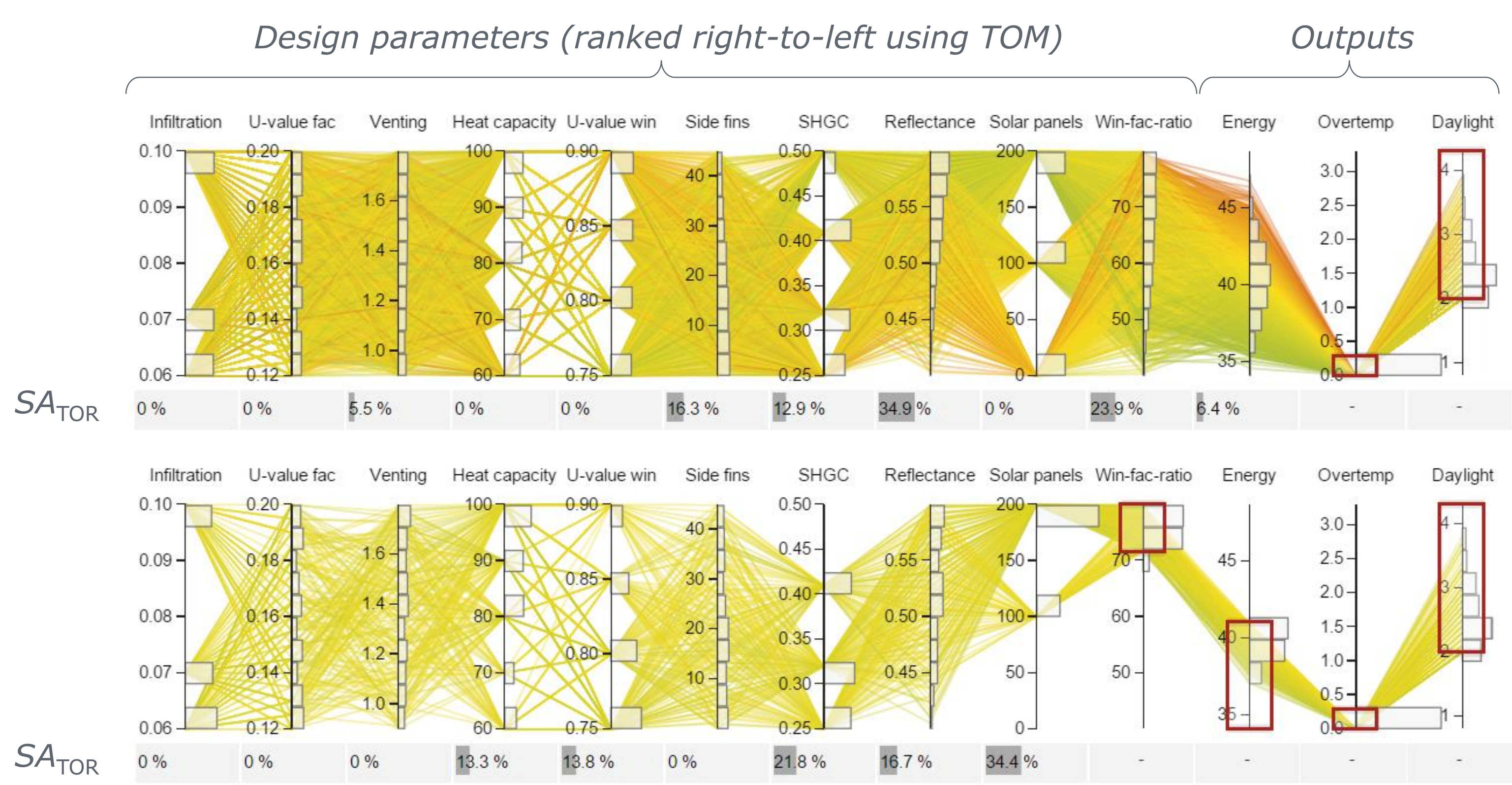
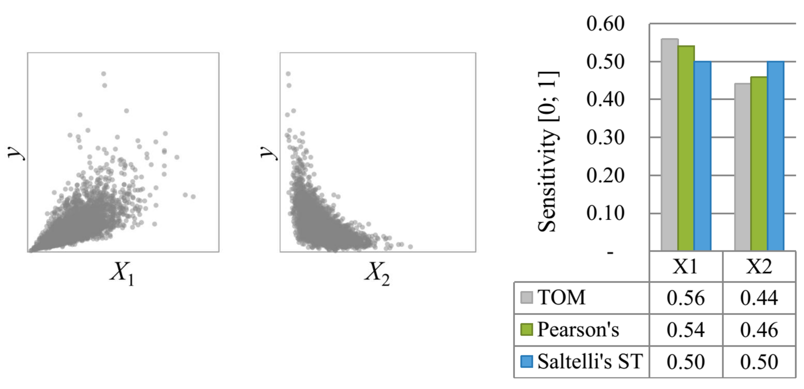


Figure 3. Screenshots of the PCP with user-defined filters (red rectangles). Grey bar plots indicate the relative sizes of the D_i's and thus highlight the parameters affected the most

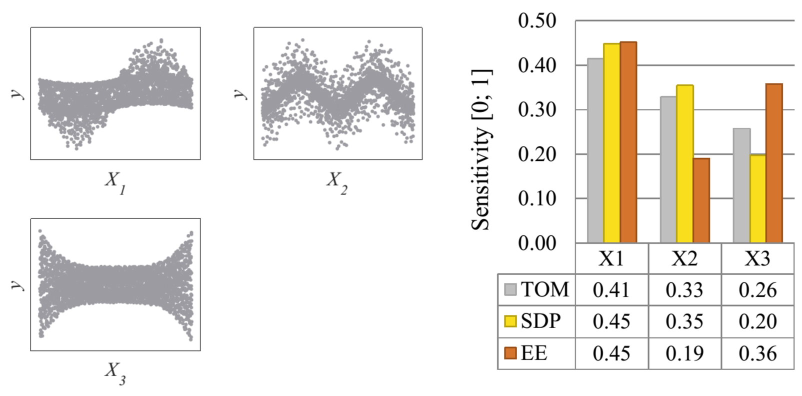
Test models (TOM)

To assess the accuracy of TOM, we apply it to three test models from literature.

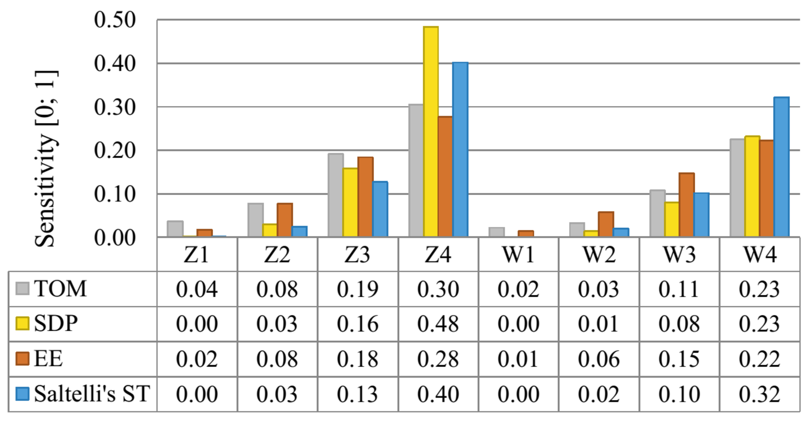
A) Highly skewed, non-linear (from [3])
$y = x_1/x_2$ (χ^2 distributions)



B) Non-monotonic, non-linear (from [4])
$y = sinX_1 + A\, sin^2 X_2 + B\, {X_3}^4 sin\, X_1$
$X_j \sim U(-\pi, \pi)$ and $A = 7, B = 0.1$



C) Non-additive (from [1])
$Y = \sum_{i=1}^{4} W_i Z_i$
$Z_i \sim N(z_i, \sigma_{Z_i}), W_i \sim N(w_i, \sigma_{W_i}),\ z_i = 0,\ w_i = ic$



Discussion

The methods seem promising for different applications. Future work includes more testing. This includes:

- More test models (non-linear)
- Tests with more inputs and outputs
- Threshold values to avoid Type I errors
- Assess choice of sets for KS2 tests
- Other statistical tests (e.g. Anderson-Darling)

Thanks to Thierry Mara for valuable feedback!

Try it yourself (TOR)

Upload your own data (tab-separated txt)
http://buildingdesign.moe.dk/phd2/html/SAMO.html

References

[1] A. Saltelli et al. (2008) *Global sensitivity analysis: the primer*. Wiley.

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[3] F. Pianosi, T. Wagener (2015) *A simple and efficient method for global sensitivity analysis based on cumulative distribution functions*. Environmental Modelling & Software.

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