Sensitivity Analysis for Model Outputs conference:
Sensivity analysis and calibration of a numerical code for the prediction of power from a photovoltaic plant

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Introduction

Context

Interests in PV

- The sun is an unlimited source of energy.
- Clean electricity production.
- High investment costs but subsequent free energy and safe.

Yield estimation issues

- Basic propagation of uncertainty + difficulties of estimating all the uncertainties → the power estimation is not precise enough.
  The financing is more and more based on a "giving the best value" method.
- Statistic and stochastic estimation with a calibrated code → provide more decision elements.

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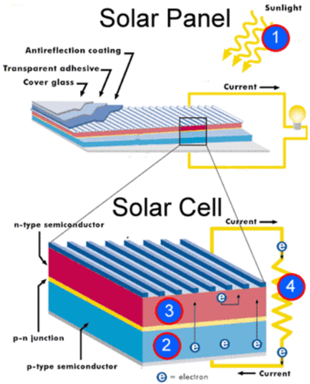
Photovoltaic

Encapsulation of a cell

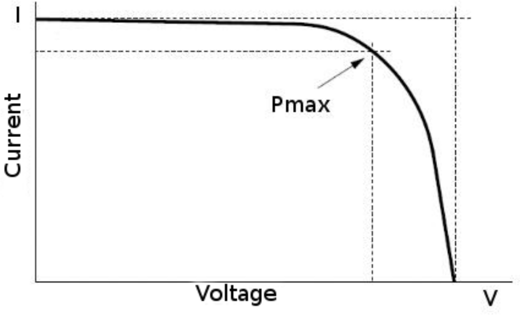


Yield losses

- Partly absorbed spectrum.
- Thermalization of the hot carriers.
- Optical reflections.
- Electrical losses in metallic contacts.
- Encapsulated in a module. $\Rightarrow$ between 15% and 20% of the light spectrum is used.

Photovoltaic

Electrical equivalences

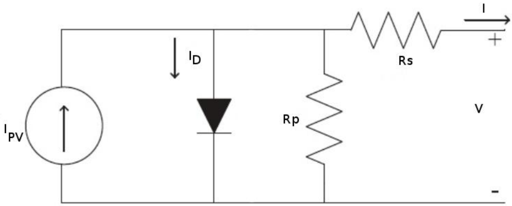


IV Curve

I and V are variables of interest $\Rightarrow$ the physical response can be represented by an electric scheme.
There is one curve for each level of irradiance and each level of temperature.

Photovoltaic

Electrical equivalences



Circuit with one diode

- A "simple" representation.
- Implicit equation can be turned into an explicit equation by approximation.
- $I = I_{PV} - I_0 \left[e^{\frac{V+IR_s}{nV_t}} - 1 \right] - \frac{V+IR_s}{R_p}$

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Computational code

Python code

Python code

- Explicit model (without any optimization).
- The output is the power.
- The inputs are from two categories (environmental variables and parameters).

Objective

Calibrate the parameters knowing the data.

Computational code

Given data

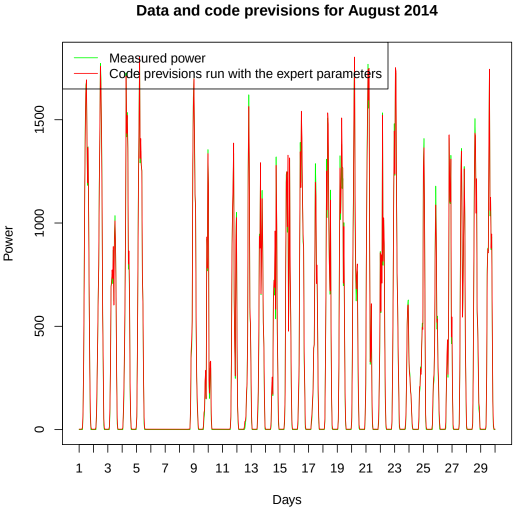
Given data

- High number of points
  - Reference stand: test facilities in activity since 2012
  - Time Step : 10s
- The power is time dependent.
- Acquisition problems may occur.

⇒ Data processing is needed.

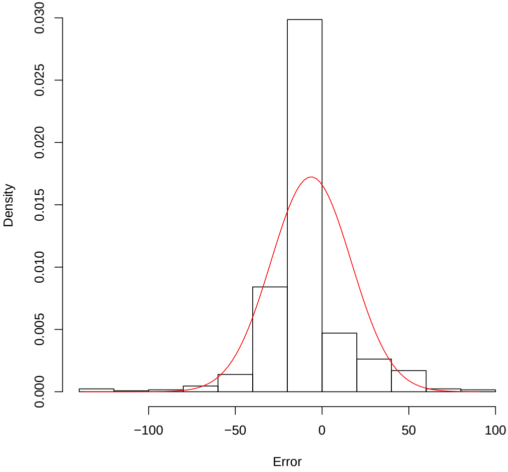
Computational code

Given data



Computational code

Code error



Computational code

Mathematical formalization [Kennedy and O'Hagan, 2001, Hidgon and al., 2005]

Code and data

$$Y_{exp_i} = f(X_i, \Theta) + \epsilon(X_i)$$

with,

$\boldsymbol{Y}_{exp}$ *measured power*

$f(X_i, \boldsymbol{\Theta})$ *computational code for environmental variables* X_i *and parameters* Θ

$\epsilon(X_i)$ *error of the acquisition measure*

$\epsilon(X_i) \sim \mathcal{N}(0, \sigma^2_{error})$

First σ_{error} *will be considered known.*

The purpose

$\Rightarrow$ Calibrate $\boldsymbol{\Theta}$ under this model.

Computational code

Parameters

Θ

$$\mathbf{\Theta} = [eta, mu, NOCT, albedo, a_r, c2, h\_conv\_vent, h\_conv, n\_inc]$$

with,

eta module photoconversion efficiency

mu module temperature coefficient (the efficiency decreases when the temperature rises)

NOCT normal operating conditions temperature of the module

albedo reflection power of the ground

a_r, c2 describe the transmission of the radiation function of the incidence angle of solar rays which depend on optical properties of the panel and the cleanliness

h_conv_vent, h_conv convection coefficients

n_inc transmission factor for normal incidence

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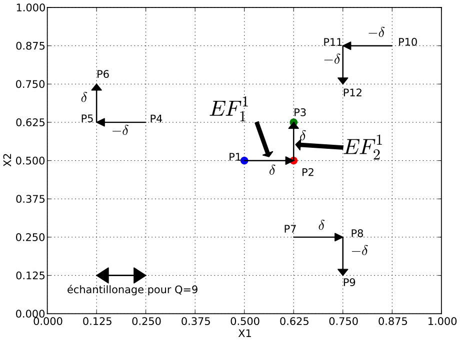
Sensitivity analysis

Morris' method [Morris, 1991]



Elementary effects

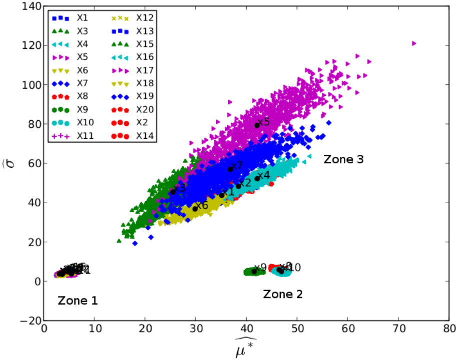
- $EF_1^1 = \frac{f(P2)-f(P1)}{\delta}$
- $EF_2^1 = \frac{f(P3)-f(P2)}{\delta}$

Indices

- $\widehat{\mu_i} = \frac{1}{R}\sum_{r=1}^{R} EF_i^r$
- $\widehat{\mu_i^*} = \frac{1}{R}\sum_{r=1}^{R} \mid EF_i^r \mid$
- $\widehat{\sigma_i}^2 = \frac{1}{R-1}\sum_{r=1}^{R}(EF_i^r - \widehat{\mu_i})^2$

Sensitivity analysis

Morris' method [Morris, 1991]

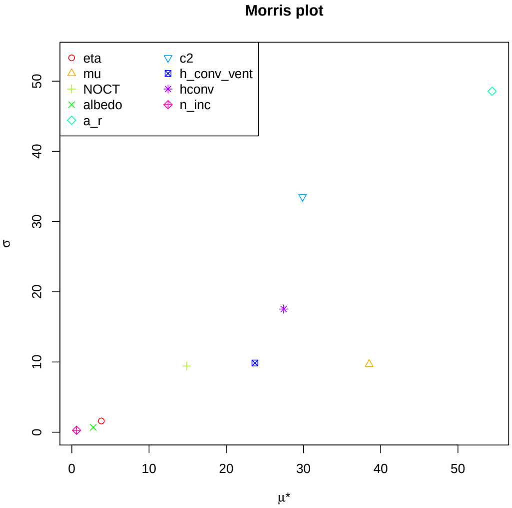


Areas

- Zone 1 : variables have an insignificant effect;
- Zone 2 : variables have linear effects without interactions;
- Zone 3 : variables have non linear and/or interaction effects.

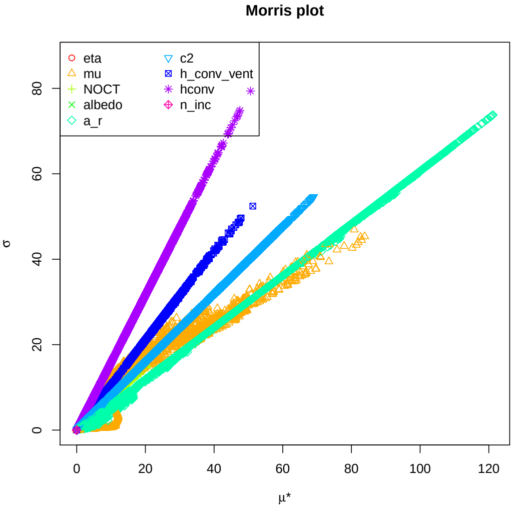
Sensibility analysis

Morris' method on the code for 25/08/2014 at noon



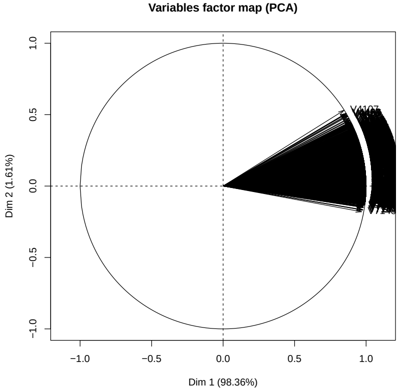
Sensibility analysis

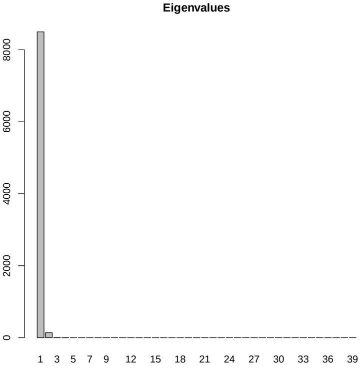
Morris' method on the code for 25/08/2014



Sensibility analysis

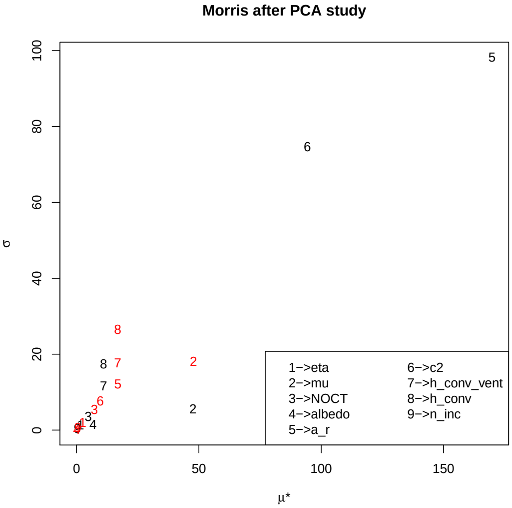
Morris' method and PCA





Sensibility analysis

Morris' method and PCA



Sensitivity analysis

Sobol indices [Sobol, 1990]

Sobol indices: Main effect and total effect

$$s_i(X_i) = \frac{\mathbb{V}[\mathbb{E}[\boldsymbol{Y}|X_i]]}{\mathbb{V}[\boldsymbol{Y}]}$$

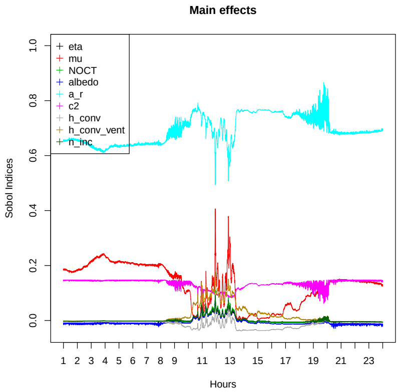
$$s_i^T(X_i) = \sum_{I \supseteq i} S_I$$

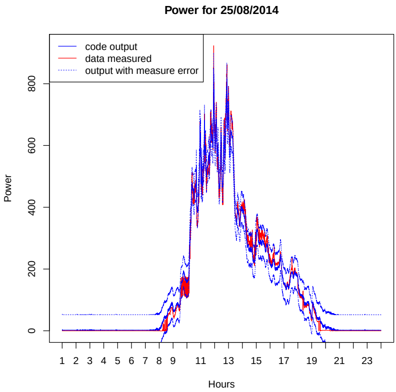
Sobol indices in a computational problem

$$s_i = \frac{\mathbb{V}[\mathbb{E}[f(\boldsymbol{X}, \boldsymbol{\Theta})|\Theta_i]]}{\mathbb{V}[f(\boldsymbol{X}, \boldsymbol{\Theta})]}$$

Sensitivity anlysis

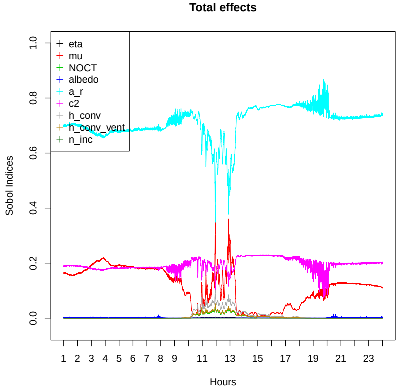
Sobol' main indices

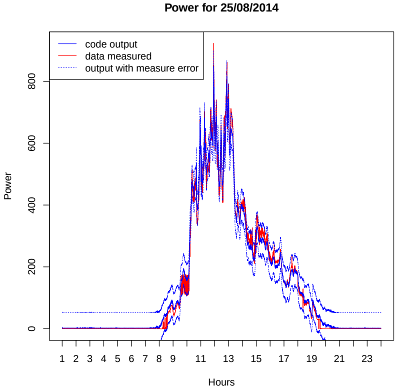




Sensitivity anlysis

Sobol' total indices





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Calibration

Bayesian framework [Kennedy and O'Hagan, 2001, Hidgon and al., 2005]

Prior

$$\boldsymbol{\Theta} \sim \mathcal{N}(\boldsymbol{\Theta}_{prior}, \Sigma_{prior})$$
$$\Downarrow$$
$$\pi(\boldsymbol{\Theta}) \propto exp\{-\frac{1}{2}(\boldsymbol{\Theta} - \boldsymbol{\Theta}_{prior})^T \Sigma_{prior}^{-1} (\boldsymbol{\Theta} - \boldsymbol{\Theta}_{prior})\}$$

Likelihood

$$Y_{exp_i} = f(X_i, \boldsymbol{\Theta}) + \epsilon(X_i)$$
$$\Downarrow$$
$$\pi(\boldsymbol{Y}_{exp}|\boldsymbol{\Theta}) \propto exp\{-\frac{1}{2\sigma^2_{error}}||f(\boldsymbol{X}, \boldsymbol{\Theta}) - \boldsymbol{Y}_{exp}||^2\}$$

Posterior

$$\pi(\boldsymbol{\Theta}|\boldsymbol{Y}_{exp}) \propto$$
$$exp\{-\frac{1}{2}((\boldsymbol{\Theta} - \boldsymbol{\Theta}_{prior})^T \Sigma_{prior}^{-1} (\boldsymbol{\Theta} - \boldsymbol{\Theta}_{prior}) - \frac{1}{\sigma^2_{err}}||f(\boldsymbol{X}, \boldsymbol{\Theta}) - \boldsymbol{Y}_{exp}||^2)\}$$

Calibration

Laplace's approximation

Laplace's approximation

The Laplace's approximation is a way to approach the *a posteriori* distribution by $\boldsymbol{\Theta}_{posterior} \sim \mathcal{N}(\hat{\boldsymbol{\Theta}}_{MAP}, \hat{\Sigma})$

- $\hat{\boldsymbol{\Theta}}_{MAP}$ is an estimation of the maximum *a posteriori* obtained by maximizing $L(\boldsymbol{\Theta}|\boldsymbol{X}) \times \pi(\boldsymbol{\Theta})$
- $\hat{\Sigma} = (-H(\hat{\boldsymbol{\Theta}}_{MAP}))^{-1}$ where H stands for the Hessian matrix $H_{i,j}(\boldsymbol{\Theta}) = \frac{\partial^2 log(\pi(\boldsymbol{\Theta})L(\boldsymbol{X}|\boldsymbol{\Theta}))}{\partial\Theta_i\partial\Theta_j}$.

Physical intervals

- $mu \in [-0.86, -0.062]$ in $\%/^\circ$C
- $a_r \in [0.1, 0.3]$

Calibration

Laplace's approximation

Expert data

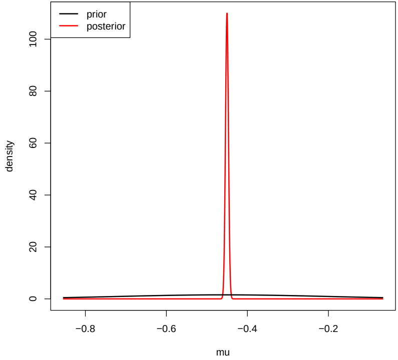
$$\mathbf{\Theta}^* = \begin{pmatrix} mu = -0.46 \\ a_r = 0.17 \end{pmatrix}$$

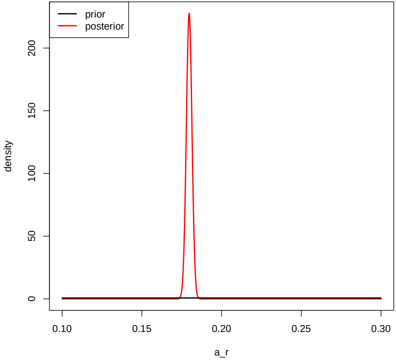
Results

$$\hat{\mathbf{\Theta}}_{MAP} = \begin{pmatrix} -0.4504 \\ 0.1797 \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} 13.1 & 3.36 \\ 3.36 & 3.07 \end{pmatrix} .10^{-6}$$

Calibration

Laplace's approximation





Calibration

Application

Algorithm

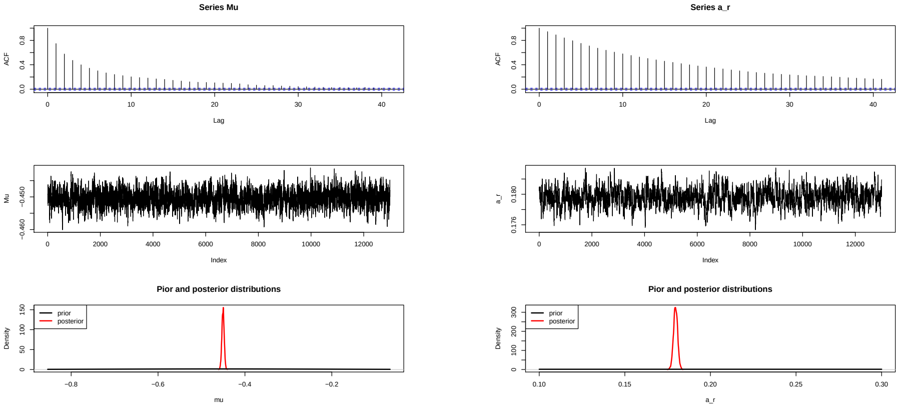
In 2 dimensions a Metropolis within Gibbs is chosen

The calibration

- Run with real data
- The physical intervals are:
  - $mu \in [-0.85, -0.0697]$ in $\%/\ ^{\circ}C$
  - $a_r \in [0.1, 0.3]$
- $k = 0.01$
- $\boldsymbol{\Theta}_{init} = \boldsymbol{\Theta}^*$
- $\Sigma_r = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ in the proposal space

Calibration

Application



Calibration

Results for the calibration

Results from Laplace's approximation

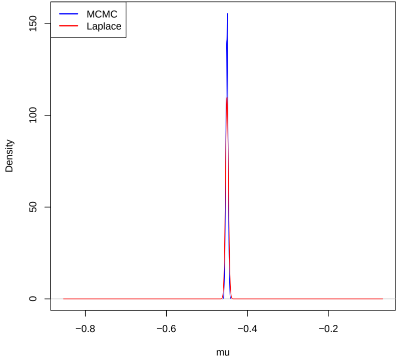
$$\hat{\Theta}_{MAP} = \begin{pmatrix} -0.4504 \\ 0.1797 \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} 13.1 & 3.36 \\ 3.36 & 3.07 \end{pmatrix} .10^{-6}$$

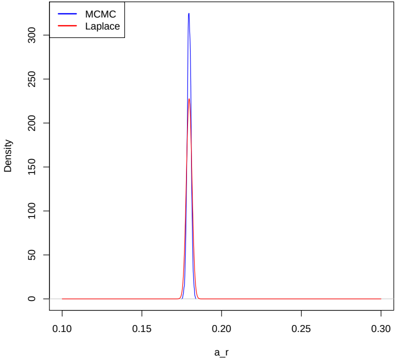
Results from the MCMC

$$\hat{\Theta}_{MCMC} = \begin{pmatrix} -0.4514 \\ 0.1817 \end{pmatrix} \qquad \hat{\Sigma}_{MCMC} = \begin{pmatrix} 6.92 & 1.52 \\ 1.52 & 1.34 \end{pmatrix} .10^{-6}$$

Calibration

Comparison between Laplace and the MCMC





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Perspectives

Conclusions

- The sensitivity analysis has allowed us to simplify the statistical model
- The dimension of the output has been bypassed but other approaches exist
- The calibration with the Laplace's is as good as the MCMC method but in a conservative point of view.

Perspectives

- Add the measurement error to the set of parameters and calibrate it with the others
- Validate the computer code
- Explore linear aspects
- Move forward to a bigger computational code which would probably require a surrogate model
- Keep in mind two objectives:
  - Give the uncertainties of the production over a long time production
  - Develop a decision making tool for the investors

References

Hidgon, D. and al. (2005).
Combining fielf data and computer simulations for calibration and prediction.

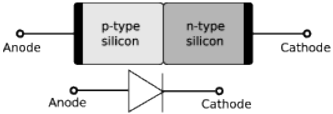
Kennedy, M. and O'Hagan, A. (2001).
Bayesian calibration of computer models.

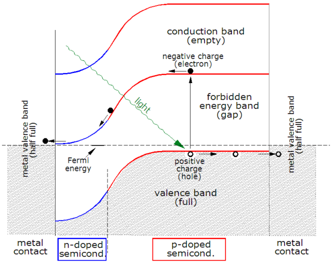
Morris, M. D. (1991).
Factorial sampling plans for preliminary computational experiments.

Sobol, I. (1990).
On sensitivity estimates for nonlinear mathematical models.

Photovoltaic

Presentation of a cell



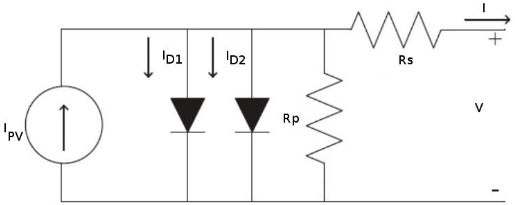


Photovoltaic effect

1. The photon arrives on the cell.
2. The energy levels are modified.
3. An electron gap pair is created if the energy in the photon is large enough.
4. This displacement leads to a movement of an electron. $\Rightarrow$ Creation of a current.

Photovoltaic

Electrical equivalences



Equation with two diodes

$$I = \frac{1+\alpha_t(T_{cell}-T_{STC})I_{sc}E}{E_{STC}N_p} - c_s S_{cell} T_{cell}^3 e^{-\frac{E_g}{V_t}} \left[e^{\frac{V+IR_s}{V_t}} - 1\right] - c_r S_{cell} T_{cell}^{5/2} e^{-\frac{E_g}{2V_t}} \left[e^{\frac{V+IR_s}{2V_t}} - 1\right] - \frac{V+IR_s}{R_p}$$

Circuit with two diodes

- Provides a better representation than the previous model.
- The equation is implicit.