Sobol' indices and variance reduction diagram estimation from samples used for uncertainty propagation

Abed Benaichouche and Jérémy Rohmer

BRGM, Orléans (France), a.benaichouche@brgm.fr

Motivations and results:

Limitations:

- The number of model runs can be very large; proportional to number of sources.
- Specific sampling schemes, available data cannot be re-used.

The proposed approach:

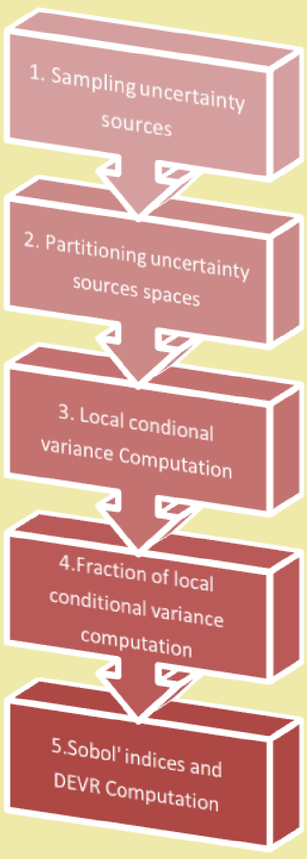
- Starting from the local conditional variance to derive the global sensitivity indicator by estimating the variance of the conditional expectation. $S_i = \frac{V[E[(Y|x_i)]]}{V[Y]} = 1 - \frac{E[V[(Y|x_i)]]}{V[Y]}$

Advantages:

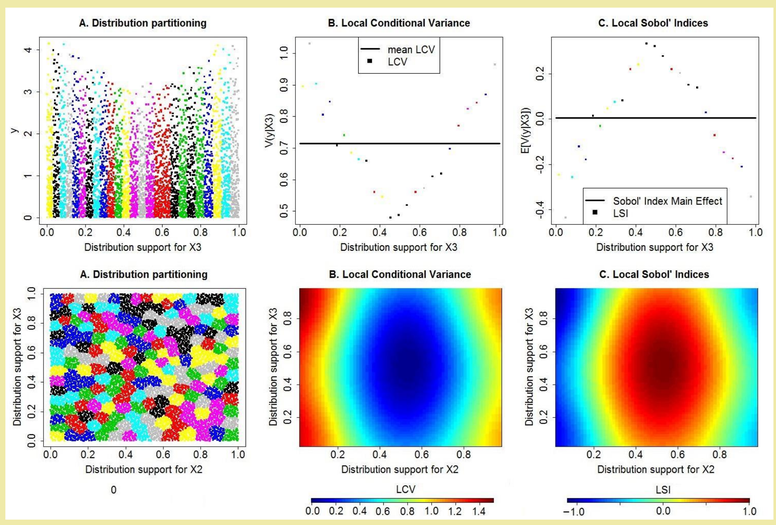
- Estimate SI without any restriction on the type of sampling scheme.
- summarize the local influence in the form of a diagram of expected variance reduction.
- Efficient comparing to classical methods.

Method description:

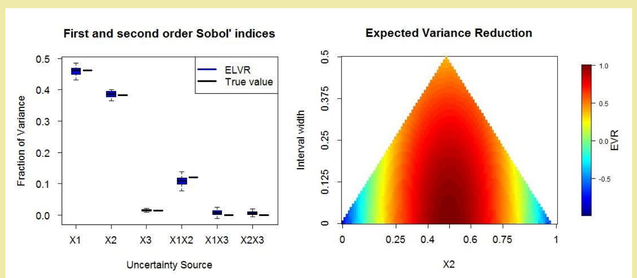
G-function: $y = \prod_{i=1}^{3} \frac{|4x_i - 2| + a_i}{1 + a_i}, where\ a_i = (0,0.1,4.5)\ and\ x_i \sim U(0,1)$



2. Partitioning: **3. Local variance:** **4. Local Sobol' index**



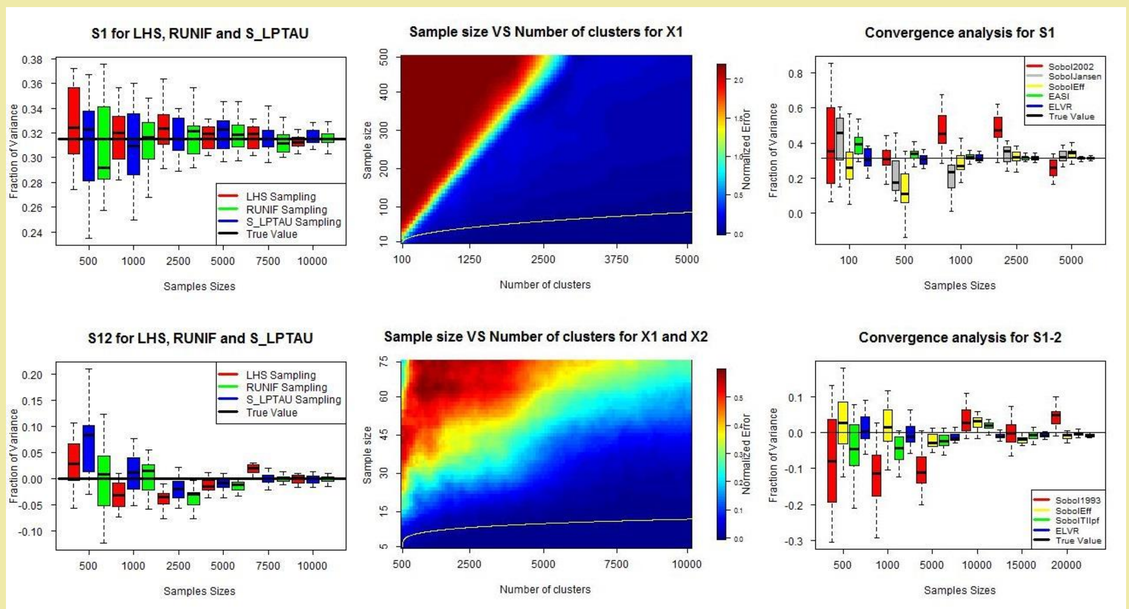
5. Sobol' indices: **DEVR for x2:**



Performance analysis on analytical cases:

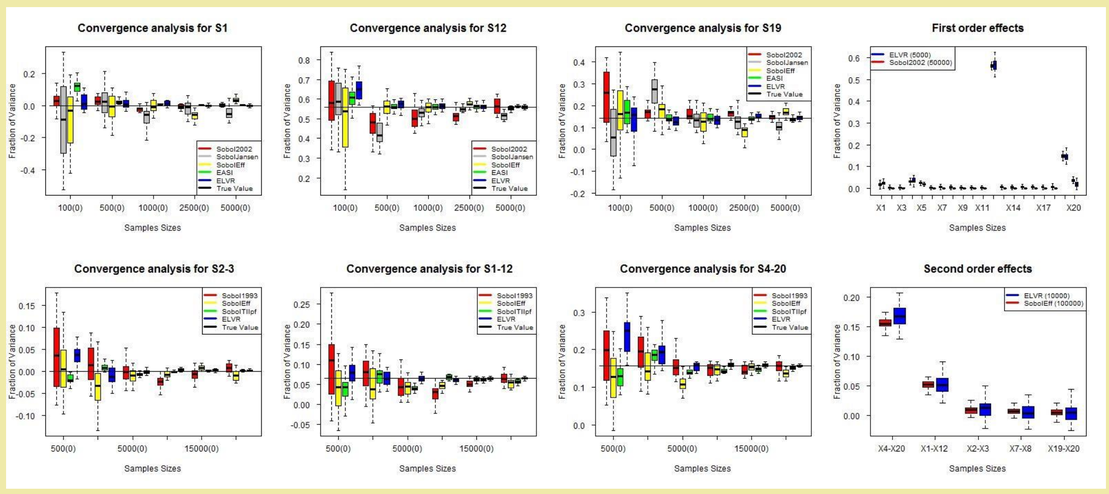
Ishigami function: $y = \sin(x_1) + 7\sin^2(x_2) + 0.1x_3^4 \sin(x_1),\ \ where\ x_i \sim U(-\pi, \pi)$

Sampling methods: **Partitioning:** **Convergence analysis:**



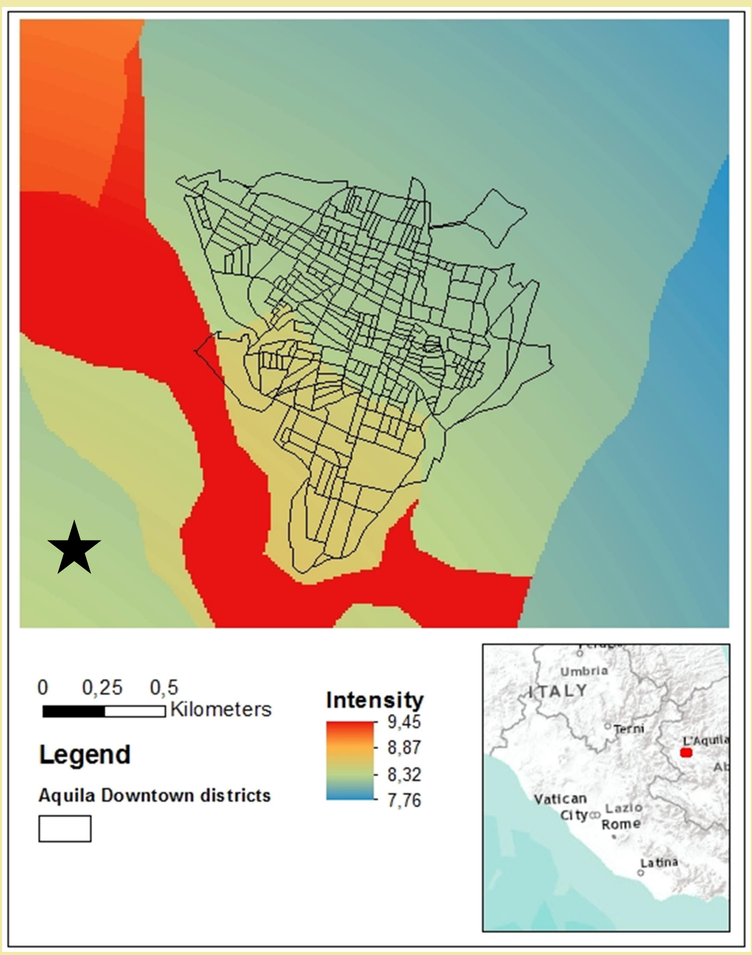
Welch function: $y = \frac{5x_{12}}{1+x_1} + 5(x_4 - x_{20})^2 + x_5 + 40x_{19}^3 - 5x_{19} + 0.05x_2 + 0.08x_3 - 0.03x_6 + 0.03x_7 - 0.09x_9 - 0.01x_{10} - 0.07x_{11} + 0.25x_{13}^2 - 0.04x_{14} + 0.06x_{15} - 0.01x_{17} - 0.03x_{18}\ \ where\ x_i \sim U(-0.5,0.5)$

Convergence analysis:



Real case application (Seismic Risk Assessment) :

Description of the focus area:

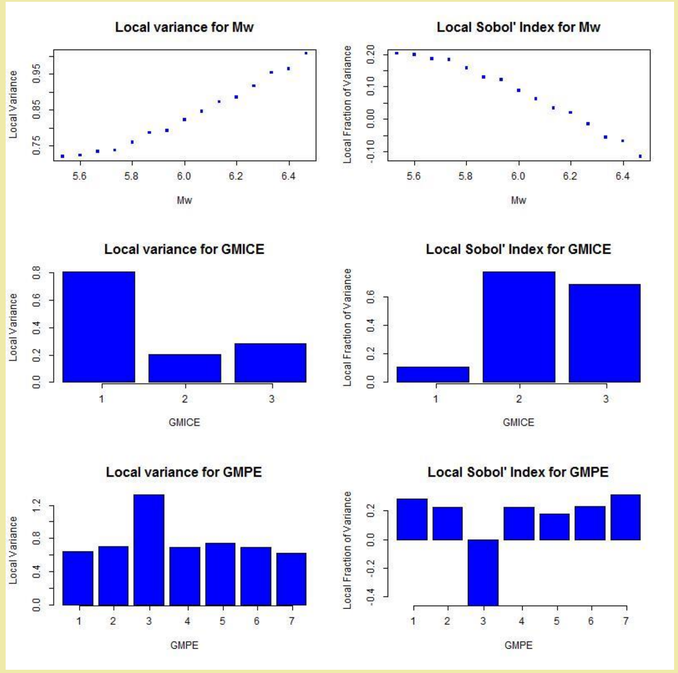


- Mw 6.3, the 6th April 2009
- Historical Aquila center area (simulation): 7,5km2
- Aquila city very impacted, but also surrounded cities
- Aquila (67000 hab), very large city
- 309 deaths
- Intensity EMS X on some city, VIII-IX in the historical center of L'Aquila.

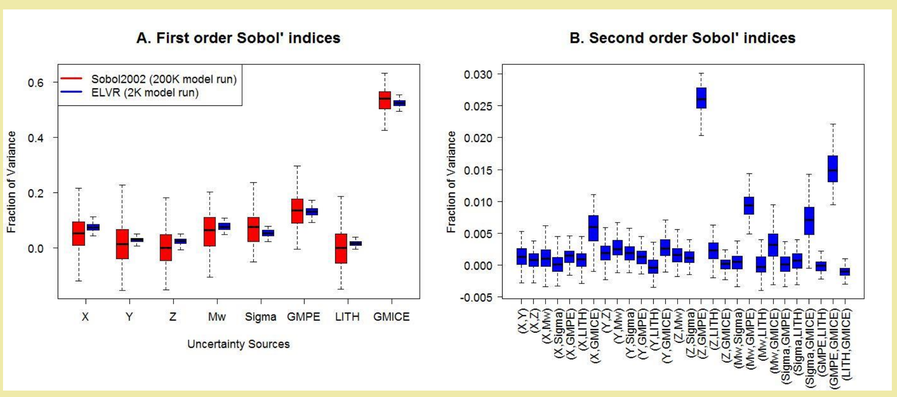
Hypothesis and uncertainty scenario:

Uncertainty source	Description and representation
XY (P)	Earthquake position U~±5km
Z (P)	Earthquake depth U~[5-15]km
Mw (P)	Earthquake magnitude U~[5,5-6,5]
Sigma (P)	Statistical parameter U~[-0,5-0,5]
GMPE (M)	Selection of the GMPE U~{1,2,3,4,5,6,7}
GMICE (M)	Selection of the GMICE U~{1,2,3}
LITH (P)	Lithological amplification ±20%

Local effects:



Main and interaction effects indices:



Conclusions:

on methodology:

- Efficient approach for SI (first and second order) calculation based on the computation of the local conditional variance calculated over a partition of the input domain space.
- Method gives local solution in the form of a diagram of sensitivities over the input space, to determine regions of input space for which the model variation is important.
- The proposed algorithms compute directly Sobol' indices from Monte Carlo samples by partitioning the input parameter space. The proposed method may potentially be hindered in regions of domain of variation where very few data are available.

on application

- Analysis of results of the first order effects shows that the model is additive (sum of seven main effects is 95.6% (no interaction between parameters).
- The most influential input corresponds to the choice of the GMICE model with a main effect of 58% followed by the GMPE model with a main effect of 14%. Therefore, an appropriate choice of GMICE and GMPE should be prioritized in future investigations.
- Additional information are given from the local analysis:
  - GMICE is fixed to {2}, the total variance on intensity could be reduced up to 80%.
  - GMICE is fixed to {1}, the total variance on intensity could be reduced up to 22%.
  - GMPE is fixed to {3}, the total variance increase by 40%

A. Benaichouche, J. Rohmer, D. Monfort Climent and C. Bellier, 2016. Identifying global and local parametric controls and dependencies using results from uncertainty propagation Reliability Engineering & System Safety (submitted).

Abed BENAICHOUCHE – benaichouche.abed@gmail.com