Expected improvement method in functional model for automotive fan design.

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Plan

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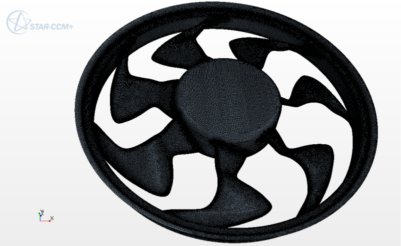
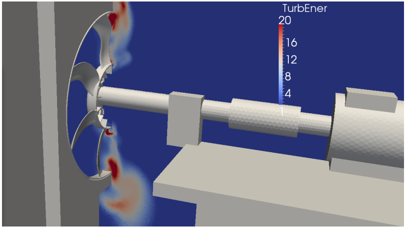
Our problem is to optimize the geometry of automotive fans shapes.





Many parameters

Chord, stagger angle, camber, sweep at some position, flowrate . Between 15 and 60 parameters . Three responses

- torque
- the difference of pressure ΔP
- the Efficiency which is a function of the two others because the speed is fixed.

The first study has consist to explore space filling designs. It was conducted in a situation of 300 points in $[0,1]^{15}$.
Basically the classicaloptimized LHS as in the Dice-kriging toolbox gave result rather better than Orthogonal arraysand determinantal processes.
This last tool is way to construct "repulsive point processes" that are defined using a kernel based on determinants. They tend to be well spread in high-dimension spaces.
Unfortunately the performance in several space-filling criterions was a little smaller than optimized LHS.

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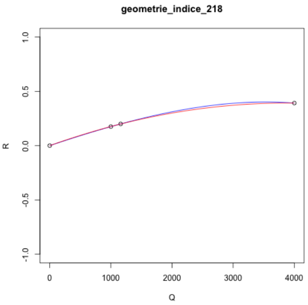
Sequential designs : position of the problem

Depending of the demand of the firm, the automotive fan can have particular specifications in terms of size or flow-rate, for example.

But in any case the efficiency is a relevant parameter. In other words, in the considered domain, many different geometries correspond to high efficiency. With respect to a specification of a given automotive firm the Efficiency will be optimized is some specific sub-domain.
The idea is to consider one of the 15 entries of the design : the flow-rate as a special variable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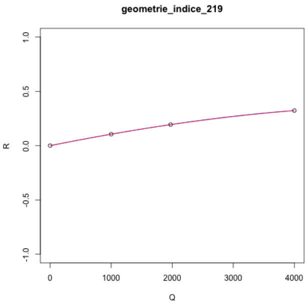
0.0
-0.5
-1.0
0
1000
2000
3000
4000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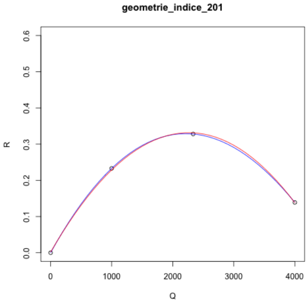
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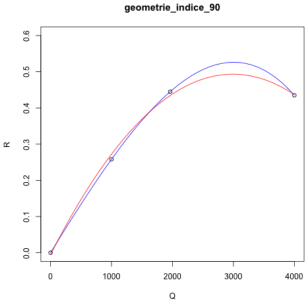
4000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0.4
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0.6
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1000
2000
3000
4000

The relevant variables

- The E_{Max}. The (maximal) efficiency at the optimal flow-rate is the natural variable
  The classical road-map is to use a kriging interpolation and then to use this interpolation to chose new point using classical methods a Expected improvement or Upper confidence bound (see later) .

  Unfortunatjly the E_{Max} is very variable and the kriging has a low performance ($R^2 \simeq 0.4$) on a validation sample.
- So we decided, though they were not the most relevant variables, to use E_{2500} the efficiency at the medium flow-rate and E_{4000} the efficiency at the maximal flow-rate before entering in the road map.

The kriging model

Kriging is performed in a high dimension space : $[0,1]^{14}$ using a separable Matern kernel. We performed simple Kriging with a simple constant mean.

$$Y(x) = m + Z(x)$$

with $Z(.)$being a centred stationary Gaussian process with a separable Matern 5/2 covariance

$$Cov(Z(x), Z(x')) = \sigma^2 \prod_{j=1}^{14} \rho_{\theta_j}(\mid x_j - x_j' \mid);$$

$$\rho_\theta(\mid x - x \mid) = (1 + \frac{\sqrt{5} \mid x - x' \mid}{\theta} + \frac{5 \mid x - x' \mid}{3\theta^2} exp(-\frac{\sqrt{5} \mid x - x' \mid}{\theta}).$$

The unknown parameter are m, σ^2 and the fourteen θ_j. That's a lot ! So in a second step we can assume that the θ_j are constant with some blocs in $\{1, \dots, 14\}$.

The expected improvement method

A new point of the design is a good point if its response can be high. This is the case if the expectation of the kriging model is high or if the variance is high. To combine these information we use a convenient criterium.

If the design consists of n points $x_1, \dots, x_n$, with response $f(x_1), \dots, f(x_n)$, if $Y(.)$ is a representation of the conditional distribution given by the kriging, we choose the next point x_{n+1} which maximises

$$\mathbb{E}\big([Y(t) - \max\{f(x_1), \dots, f(x_n)\}]^+\big).$$

It is an exercice to compute that if Y follows a $N(\mu, \sigma^2)$ then

$$\mathbb{E}(Y^+) = \sigma\phi(\mu/\sigma) + \mu\Phi(\mu/\sigma)$$

so the function to maximize is explicit and the maximisation is easy...

Batch computation

We can, of course, compute the response $f(x_{n+1})$ at the new point and redo everything : kriging, maximization to compute another new point. But this is very heavy, so we prefer to compute a batch of several points say b.
The new criterium becomes

$$\mathbb{E}\big([\max\{Y(x_{n+1}),\ldots,Y(x_{n+b})\} - \max\{f(x_1),\ldots,f(x_n)\}]^+\big).$$

The computation is now more complex and can be performed using tool for integration of multivariate Gaussian process (including Monte-Carlo)

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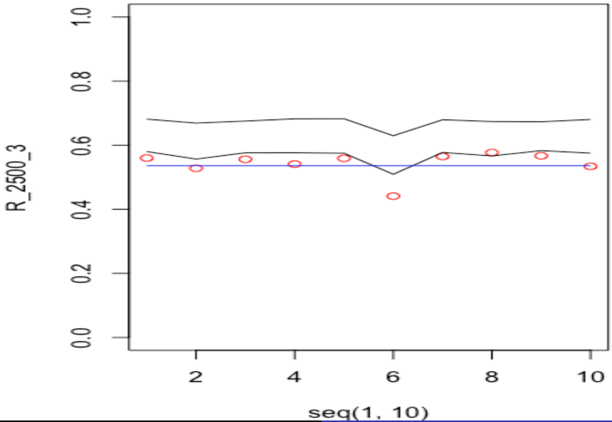
20 new geometries have been proposed at flow-rate 2500

Actual Efficiency of the new Expected Improvement points

Confidence intervals in the kriging model.

Former maximal Efficiency

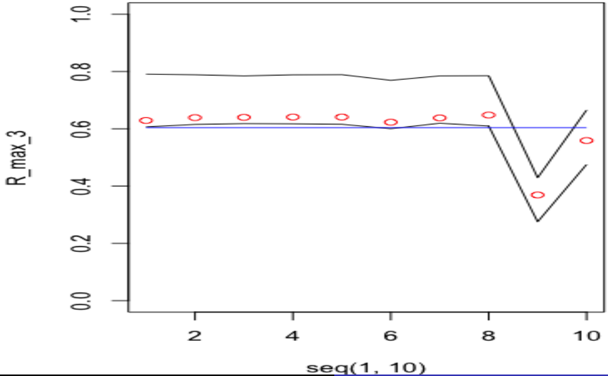


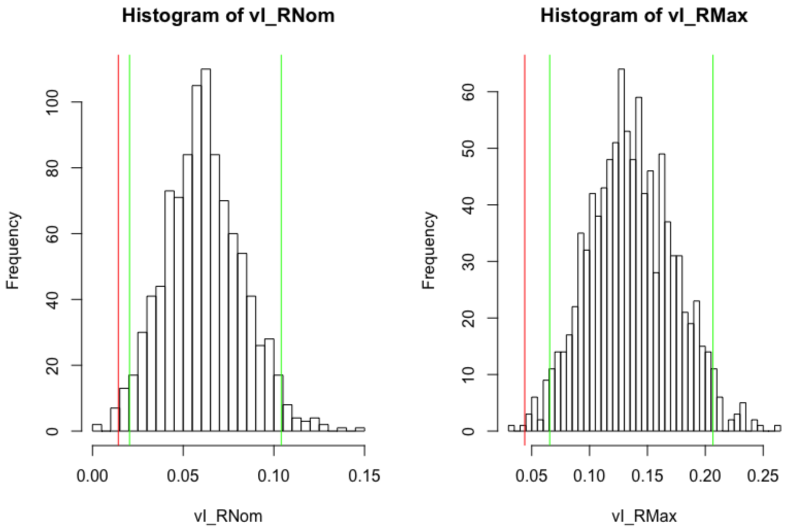
Efficiency at 4000 Flowrate R_{4000}

Actual Efficiency of the new Expected Improvement points

Confidence intervals in the kriging model.

Former maximal Efficiency




Histogram of vl_RNom
Frequency
0
20
40
60
80
100
0.00
0.05
0.10
0.15
vl_RNom
Histogram of vl_RMax
Frequency
0
10
20
30
40
50
60
0.05
0.10
0.15
0.20
0.25
vl_RMax

Conclusion

- The functional model has permitted to construct relevant additional points.
- Optimization in the batch method is in progress
- UCB :Upper confidence bound method has to be compared to Expected Efficiency
- Larger dimension must explored

MERCI!

THANK-YOU

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