Sensitivity analysis under different distributions using the same simulation with an application in the nuclear sector.

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EXTENDED ABSTRACT

Modelling is used in many areas of science and technology to deal with safety and security problems. When dealing with this type of problems, risk / safety / security criteria are established and the technical body of the organization developing the study has to prove compliance of the system under study with the criteria (think, for example, of nuclear reactor safety, high level nuclear radioactive waste repositories safety assessments or security of gas supply). Criteria are typically established on some output variable, such as for example the dose to the population or the quantity of unserved gas. Most frequent criteria are based either on the expected value of the output variable or on some probability of exceedance (the expected dose should not exceed a given reference value, or the probability of not satisfying the demand of gas protected customers at least one day of the year should be less than 5%).

In many of these problems, the uncertainty in the input parameters is characterized by means of probability density functions (pdf) via expert judgement, given the intrinsic difficulty, even impossibility of taking actual measurements. When this happens, sometimes Sensitivity Analysis (SA) practitioners and analysts are confronted with the problem of having different, conflicting pdfs for characterizing the uncertainty in a given input parameter. Sometimes experts acknowledge large uncertainty in the scale and shape of the input parameters; sometimes they provide conditional probabilities (dependent on other uncontrolled parameters); sometimes they evendo not agree. Even if pdfs are obtained via actual measurements, pdfs may evolve over time after the acquisition of new information (Bayesian update of information). Under these circumstances, SA practitioners are asked how the output variables distributions could change given changes in the input parameters pdfs, and in particular, if alternative input parameters pdfs could deliver significant changes in terms of criteria violation.

During the last two decades we have seen a huge development in the area of SA (variance based techniques, Monte Carlo filtering, graphical techniques, etc.), but this problem has been systematically ignored in the literature. The trivial solution to this problem is to execute Monte Carlo with the default input multivariate distribution ($f_1(x)$) and to repeat it again, using a new sample, using the alternative ($f_2(x)$) pdf, comparing afterwards the statistics obtained and apply the corresponding test to determine if differences are statistically significant (provided that the adequate test exists). Certainly this procedure is far from optimal.

McKay and Beckman [1] are among the few authors that have addressed formally this problem. These authorspropose two methods to estimate the effect of changing the input distributions: the rejection method and the weighting method. Hesterberg [2] addresses also the problem from the point of view of importance sampling. The approach adopted in this paper is not the importance sampling approach, but the SA view. The estimator proposed for the distribution of the output variable is the one that assigns a new weight

$$P(Y(\mathbf{x}_i)) = \frac{\dfrac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}{\sum_{i=1}^{n} \dfrac{f_2(\mathbf{x}_i)}{f_1(\mathbf{x}_i)}}$$

to each sampled output value Y. In this way one can estimate the pdf of the output for different pdfs of the input and, in particular, variance-based sensitivity indices for different input pdfs, using the same simulations.

We have applied this method to the assessment of the safety of a passive system in a nuclear power plant. This system is the BOPHR/RP2 "**B**ase **O**peration **P**assive **H**eat **R**emoval strategy applied to **R**esidual **P**assive heat **R**emoval system on the **P**rimary circuit. This system is considered passive because after the start of the nuclear accident it relies only of physical principles to accomplish its mission (adequately cool down the reactor core and keep moderate the pressure in the primary circuit). In particular it should be able to work with total lack of electricity supply, relying only on natural circulation. In this problem 14 input parameters affected by uncertainty were considered. The system was considered to succeed if it were able to keep pressure in the primary circuit below 4 MPascal.

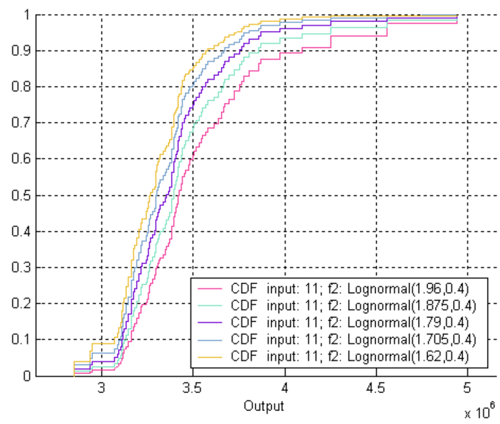

Fig. 1. Different empirical distributions of the output variable for different distributions of input parameter 1.

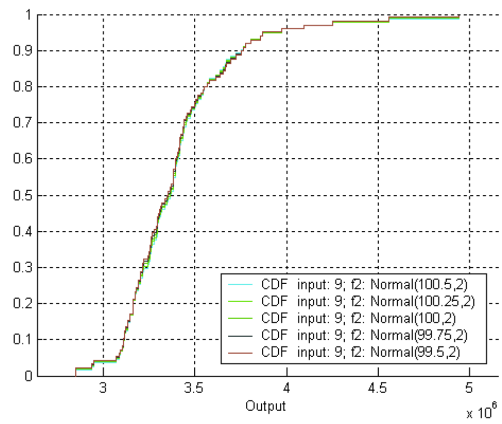

Fig. 1. Different empirical distributions of the output variable for different distributions of input parameter 2.

Figures 1 and 2 show the results obtained for the Pressure when the pdfs of two different parameters are changed. Input parameter 1 is very important, while input parameter 2 is irrelevant to the output variable considered. Under the default distribution (purple line), the estimated probability of failure of the system (pressure exceeding 4MPascal) is approx. 0.04. Under the different alternative pdfs for parameter 1, the probability of failuremay change between approx. 0.01 and 0.11. On the other hand, the probability is completely insensitive to changes in the pdf of input parameter 2. In the paper we show how to derive the proposed approach, its properties and statistical tests applicable to determine if the differences are statistically significant.

References

1. Beckman R.J., McKay M.D. 'Monte Carlo Estimation Under Different Distributions Using the Same Simulation'. **Technometrics** Vol 29, No 2, pages 153-160. 1987.
2. Hesterberg T. 'Weighted Average Importance Sampling and Defensive Mixture distributions'. **Technometrics** Vol 37, No 2, pages 185-195. 1995.