The role of Rosenblatt transformation in global sensitivity analysis of models with dependent inputs

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Abstract

Reliable methods exist to perform global sensitivity analysis (GSA) of computer models with independent input factors. In that case, there are different reliable importance measures proposed in the literature to perform this task (e.g. [1–4]). Performing GSA of models with dependent input factors is more challenging. Several has been recently introduced to perform such an analysis ([5–7]). In the cited papers, the authors introduced defined variance-based sensitivity indices for models with dependent inputs and also proposed different methods to assess them.

Following the original idea of [8], the new variance-based sensitivity indices allow for distinguishing the contributions of an input factor to the model response variance that account for its dependence with the other inputs and that do not account for its joint dependence contribution. Such a kind of distinction also stands if one consider another type of importance measure. For instance, in [9], the authors defined these kind of indices for the moment-independent measure of Borgonovo [3]. The aim of my presentation is to highlight the role of Rosenblatt transformation (RT, [10]) for analysing computer models with dependent input factors.

Let $y = f(\boldsymbol{x})$ be the model response of a computer model function of n random input factors $\boldsymbol{x} = (x_1, \ldots, x_n) \sim p(\boldsymbol{x})$. RT transforms $\boldsymbol{x}$ into a random vector $\boldsymbol{u}$ uniformly distributed over the unit hypercube $\mathbb{K}_n$. RT can be written as follows,

$$\begin{cases} u_1 = F_1(x_1) \\ u_2 = F_{2|1}(x_2|x_1) \\ \vdots \\ u_n = F_{n|\sim n}(x_n|\boldsymbol{x}_{\sim n}) \end{cases} \tag{1}$$

where F_1, $F_{i_k|i_1 \ldots i_s}$ are respectively the cumulative distribution function of $x_1 \sim p_1(x_1)$ (i.e. $p_1(x_1) = \mathrm{d}F_1/\mathrm{d}x_1$) and the conditional cumulative distribution function of $x_2 \sim p_{2|1}(x_2|x_1)$. The vector $\boldsymbol{x}_{\sim n}$ stands for $\boldsymbol{x}/x_n$.

Once the input vector $\boldsymbol{u}$ obtained, it is straightforward to compute any desired sensitivity index related to the u-variables since they are independent. The interpretation of the sensitivity of $\boldsymbol{u}$ as those of $\boldsymbol{x}$ where given first in [6] (actually, during the VI-th SAMO Conference 2010 in Milan). The sensitivity indices of u_1 are those of x_1, the sensitivity indices of u_2 are those of x_2 without its mutual influence due to its dependence with x_1, etc. Finally, we note that the influence of u_n onto the model response is simply the one of x_n without accounting for its dependence with the other variables.

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To assess the independent influence of all the inputs onto the model response, one must notice that the Rosenblatt transformation is not unique. Indeed, one can start with any input variable x_i in (1) and also ends the transformation with any x-variable. The main drawback of RT is that the conditional densities must be known in advance. But, the advantage is that one can perform GSA with any importance measure and also any method proposed in the literature. Thus, one can adapt the Morris method to the screening of computer models with dependent inputs as proposed proposed by some authors (paper under evaluation).

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