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Joint Research Centre

Weights and Measures: Exploring and Optimising Composite Indicators with Sensitivity Analysis

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A Measure of Transparency and Accountability in the Oil, Gas and Mining Sector

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Composite indicators are aggregations of observable variables that aim to quantify non-measurable concepts.

Why

- Quantification of hard-to-quantify concepts
- Aggregation of a large number of indicators into an easily-digestible format
- Simple comparisons between countries/entities

THE GLOBAL SLAVERY INDEX 2016

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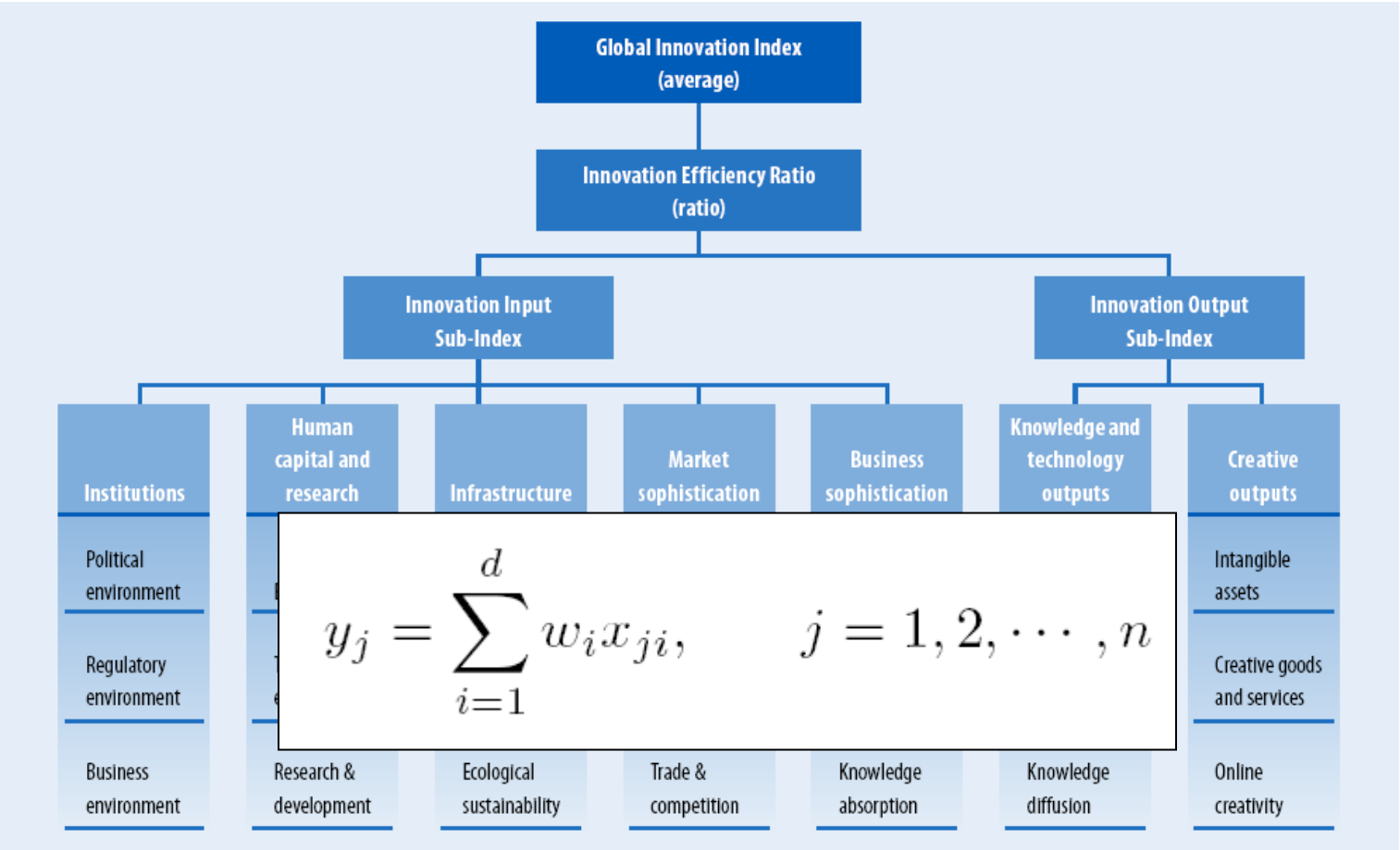

$$y_j = \sum_{i=1}^{d} w_i x_{ji}, \qquad j = 1, 2, \cdots, n$$

x_1 x_2 x_3 x_4 x_5 x_6 x_7

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Composite indicators

- ✓ Simple* aggregations of indicators
- ✓ Very subjective
- ✓ Very uncertain
- ✓ Attract lots of public, political and media attention

*Actually not simple: statistical analysis, conceptual framework, robustness checks, etc…

$$y_j = \sum_{i=1}^{d} w_i x_{ji}, \qquad j = 1, 2, \cdots, n$$

"a general criticism that is frequently addressed at composite indicators, i.e. the arbitrary character of the procedures used to weight their various components"

- the Stiglitz Report, 2010

Two ways to weight

Weights by belief

- Equal weights
- Budget allocation
- Public opinion
- Analytic hierarchy process
- Conjoint analysis

Weights by Statistical models

- Principal component analysis
- Factor analysis
- Data envelopment analysis
- Regression approach

Weights are typically assigned to reflect importance, but…

$\neq$

weights do not equal measures

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A composite indicator to rank teachers…

x_1=number of publications
x_2=teaching feedback
x_3=hours of teaching and office work

$$y = \frac{1}{3}(x_1 + x_2 + x_3)$$

All have been standardised to have unit variance, *but* x_2 and x_3 have a correlation of 0.7. After calculating the values of the composite indicator, R^2 is used to check influence:

$$R_i^2 := \mathrm{corr}^2(y, x_i)$$

R_1^2=0.227
R_2^2=0.657
R_3^2=0.657

Increase the weight of x_1? Teachers will complain that the index unfairly favours publications!

Correlations are very common in composite indicators.

$\neq$

How can we measure importance?

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Measuring the importance of indicators

Correlation coefficient

$$R_i := \mathrm{corr}(y, x_i) := \frac{\mathrm{cov}(y, x_i)}{\sigma_y \sigma_i}.$$

$$R_i^2 := \mathrm{corr}^2(y, x_i),$$

Ok but only measures linear dependence. Not always the case.

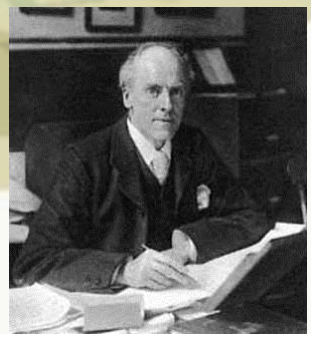
Correlation ratio*

$$S_i \equiv \eta_i^2 := \frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)},$$

Also known as "main effect index", "first-order sensitivity index", "nonlinear R^2"…

- Allows for nonlinear dependence
- Easily estimated by regression

*First conceived by Karl Pearson in 1905



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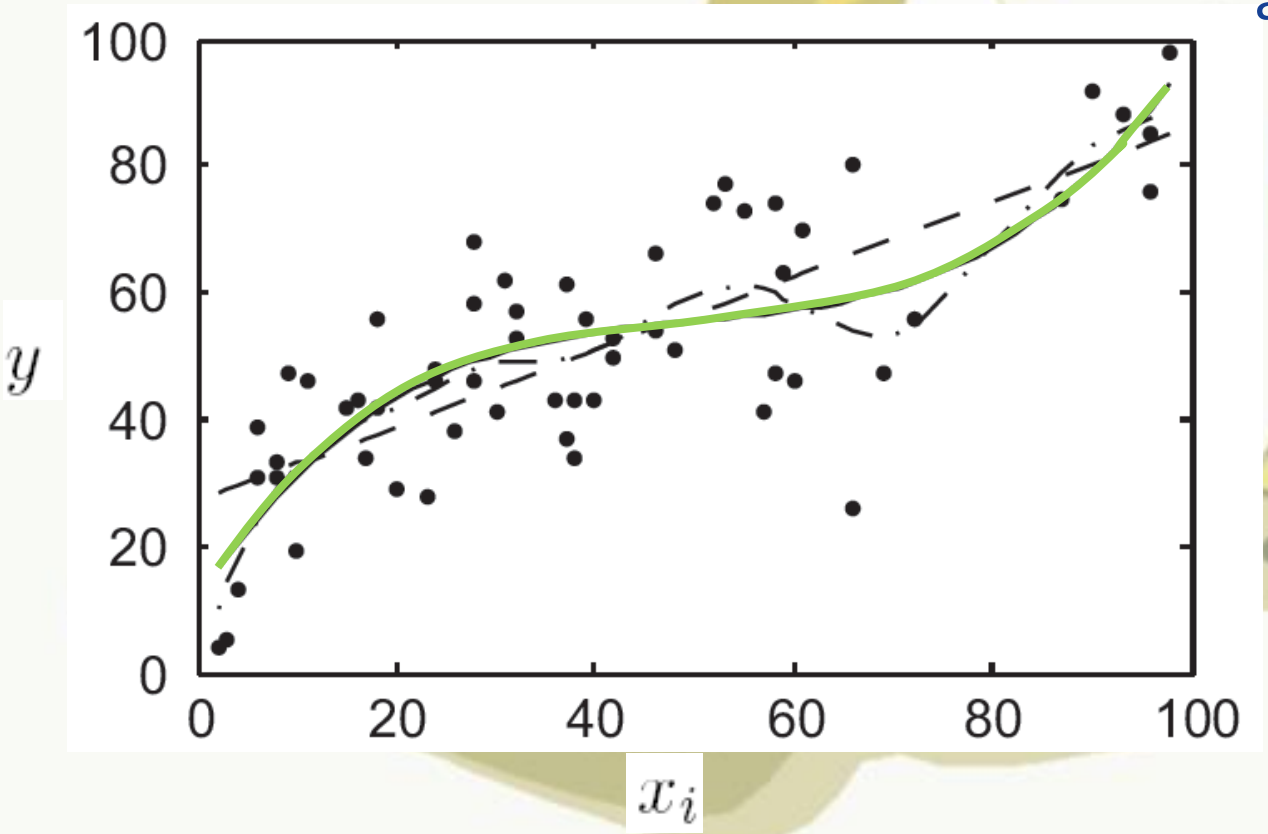
$$S_i \equiv \eta_i^2 := \frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}\sim i}(y \mid x_i)\right)}{\mathrm{V}(y)},$$

Nonlinearity in the main effect

Nonlinear regression approaches required



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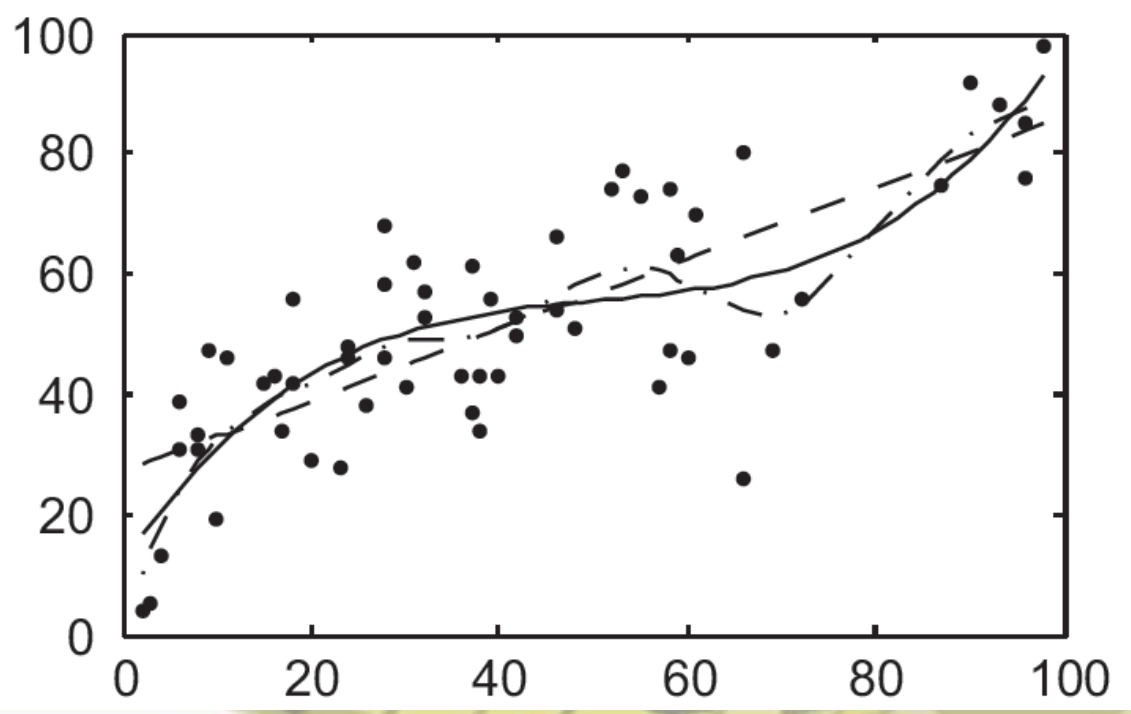
$$S_i \equiv \eta_i^2 := \frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}\sim i}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)},$$



$$\frac{\sum_{j=1}^{n}(\hat{y}_j - \bar{\bar{y}})}{\sum_{j=1}^{n}(y_j - \bar{y})}$$

$p(\mathbf{x})$ unknown:

Arbitrarily-distributed dependent variables.

Nonlinear regression approaches

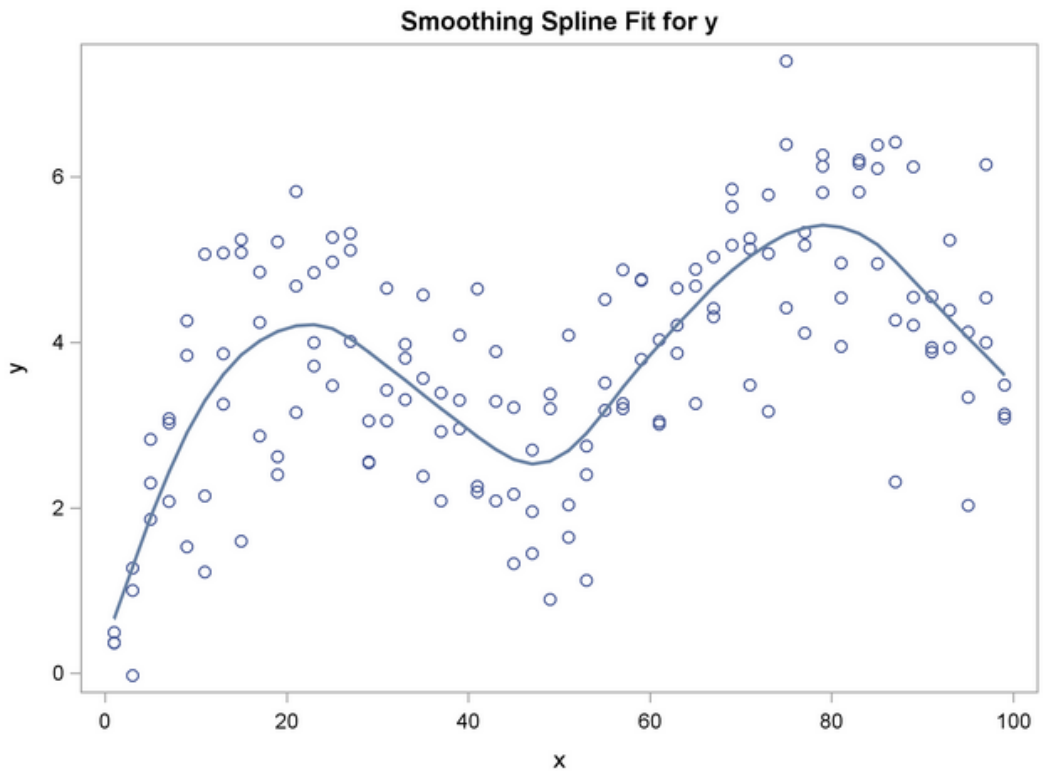
1. Penalised smoothing splines

$$\beta_0 + \beta_1 x + \ldots + \beta_p x^p + \sum_{k=1}^{K} \beta_{pk} (x - \kappa_k)_+^p$$

Polynomial part Spline part

The two components are weighted by a smoothing term

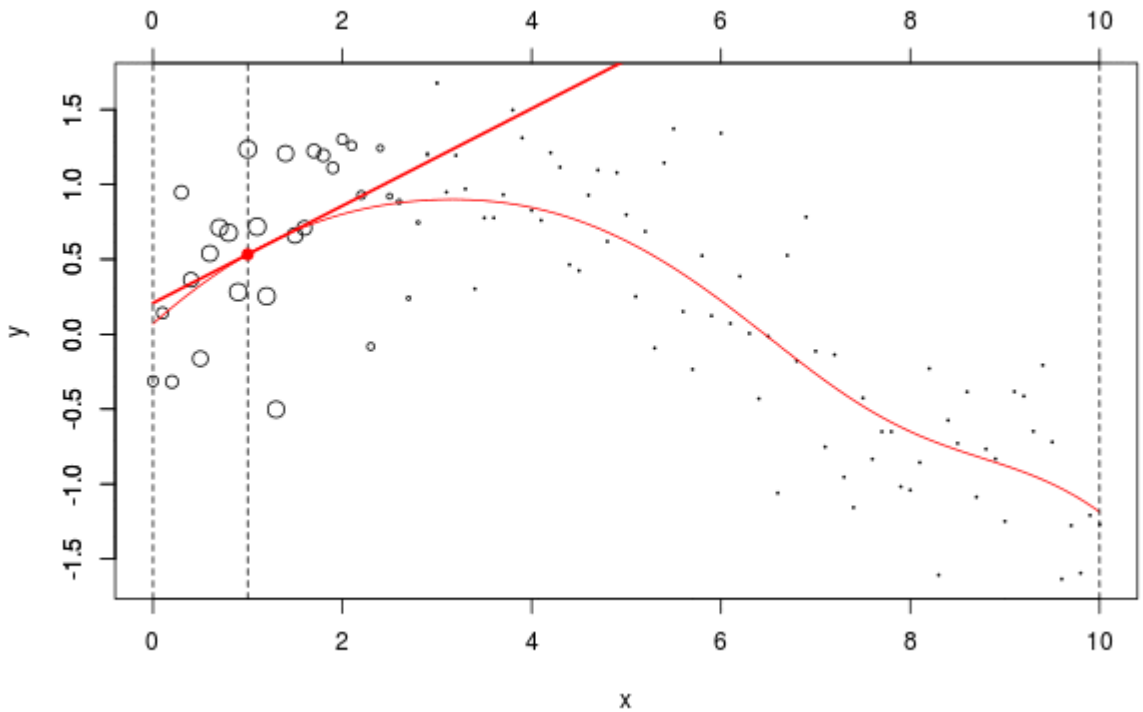
1. Penalised smoothing splines



$$\beta_0 + \beta_1 x + \ldots + \beta_p x^p + \sum_{k=1}^{K} \beta_{pk} (x - \kappa_k)_+^p$$

- Smooth fits
- Fast parameter estimation using linear regression properties: can deal with large datasets

2. Local linear regression



Source: https://www.r-bloggers.com/some-heuristics-about-local-regression-and-kernel-smoothing/

Weighted polynomial regression.

$$\hat{m}(x_i) = \frac{\sum_{j=1}^{n} w(x_{ji} - x_i; h) y_j}{\sum_{j=1}^{n} w(x_{ji} - x_i; h)}$$

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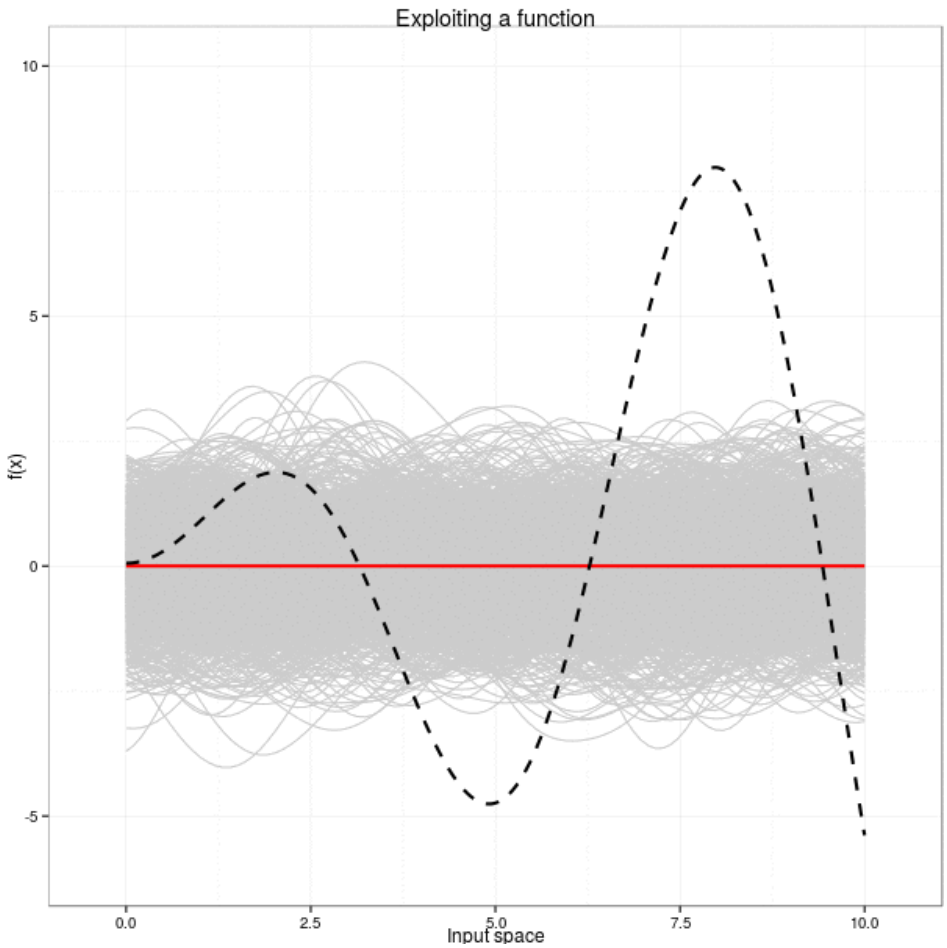
3. Gaussian process regression (kriging)



A probabilistic distribution over functions.

Points are assumed to be distributed joint-normally.

Bayesian approach to fitting: sample from posterior using MCMC.

Posterior draws can be used to build a distribution over S_i.

$$f(x)|\sigma^2, b, \sigma_n^2 \sim \mathcal{GP}\left(m(x), \sigma^2 c(x, x') + \sigma_n^2\right)$$

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Can we do more?

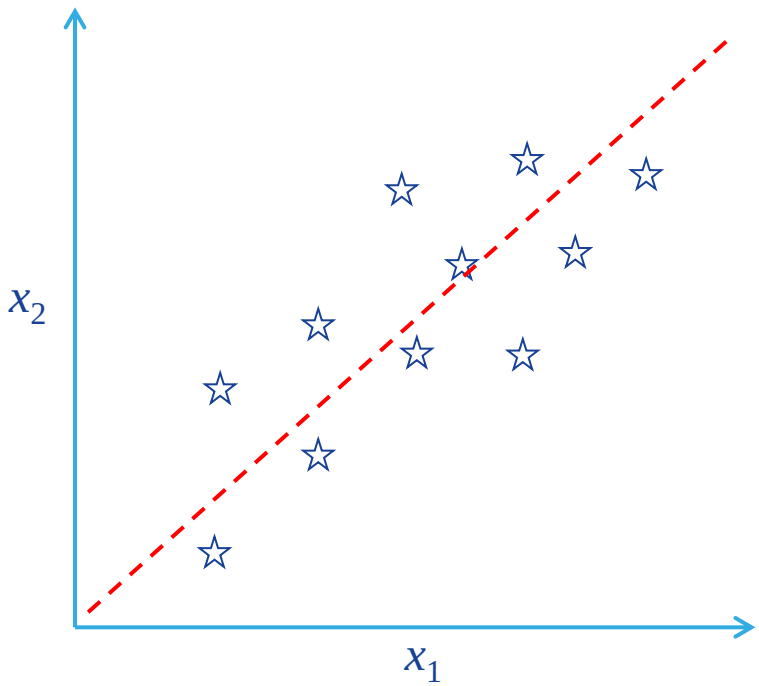
Separating correlation from aggregation

$$S_i = S_i^u + S_i^c$$

Correlation ratio Uncorrelated part Correlated part

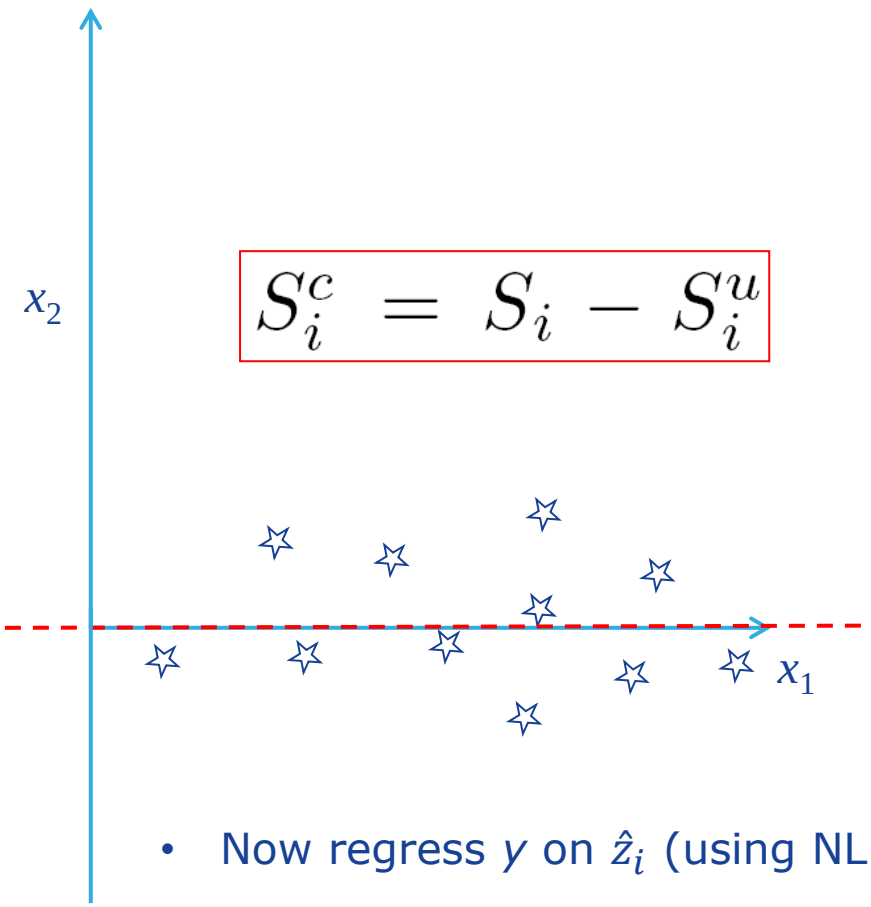
- Reveals variables that could be removed (uncorrelated contribution is low)
- Reveal indicators which are not sensitive to changes in weights

Separating correlation from aggregation



We regress x_i on to the remaining variables $x_{\sim i}$

Separating correlation from aggregation



$$S_i^c = S_i - S_i^u$$

We regress x_i on to the remaining variables $x_{\sim i}$

Then subtract from x_i: this effectively removes the dependence on other variables.

$$\hat{z}_i = x_i - \hat{x}_i = x_i - \beta_0 + \sum_{l \neq i}^{d} \hat{\beta}_l x_l$$

- Now regress y on $\hat{z}_i$ (using NL regression) to get uncorrelated part.

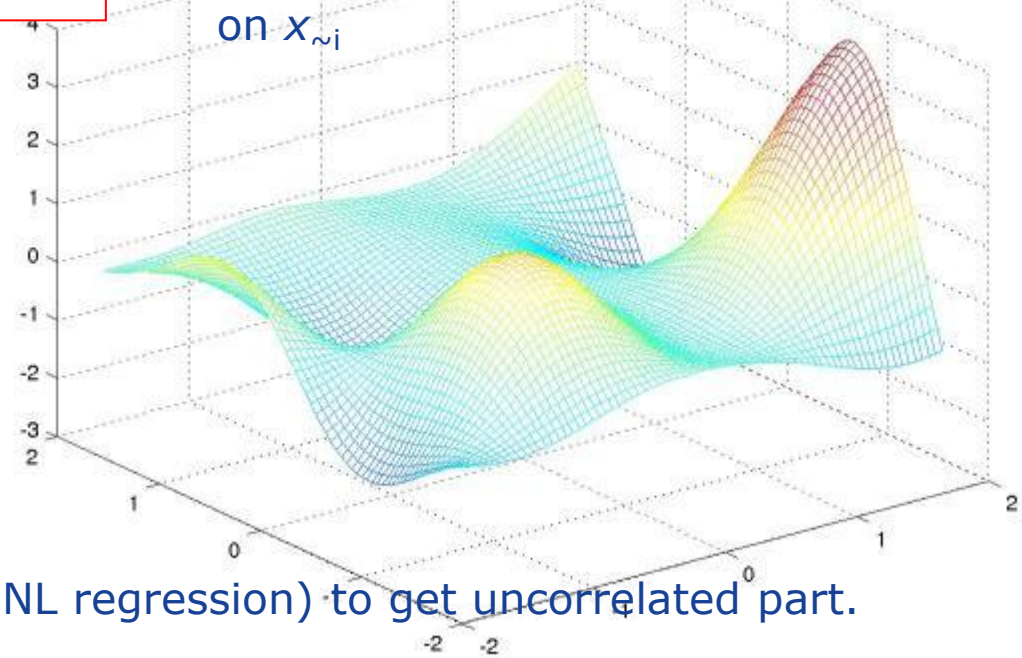
Separating correlation from aggregation

$$\hat{z}_i = x_i - \hat{x}_i = x_i - \beta_0 + \sum_{l \neq i}^{d} \hat{\beta}_l x_l$$

$$S_i^c = S_i - S_i^u$$

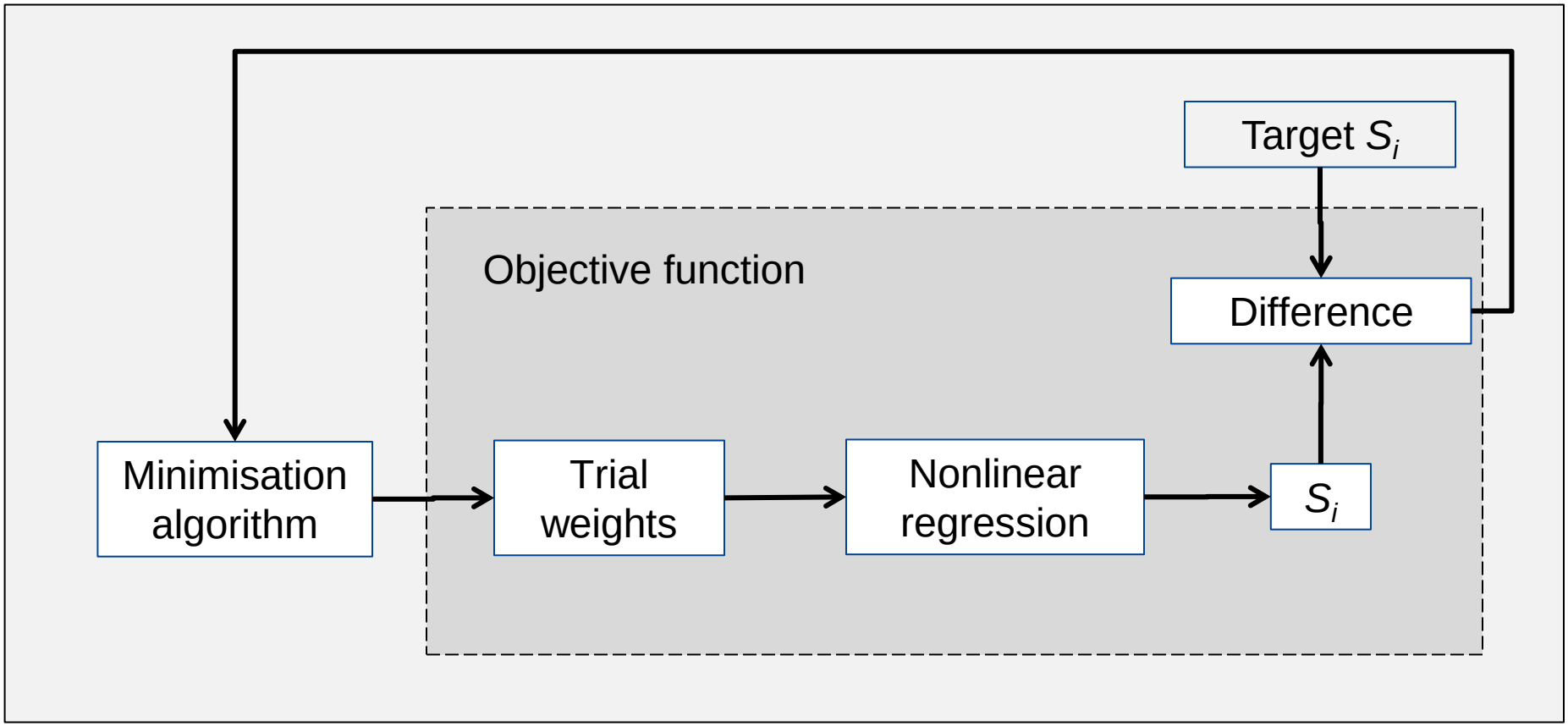
$$\frac{\sum_{j=1}^{n}(\hat{y}_j - \bar{\bar{y}})}{\sum_{j=1}^{n}(y_j - \bar{y})}$$

Gaussian process regression of x on $x_{\sim i}$



- Now regress y on $\hat{z}_i$ (using NL regression) to get uncorrelated part.
- Can also use nonlinear regression to remove nonlinear dependence

Can we tune the weights?



$$\boldsymbol{w}_{\text{opt}} = \text{argmin}_{\boldsymbol{w}} \sum_{i=1}^{d} (\tilde{S}_i^* - \tilde{S}_i(\boldsymbol{w}))^2.$$

Summary

- Estimate S_i of indicators using nonlinear regression (allows for correlation)
- Separate correlated part of S_i using a regression approach (generalised to nonlinear dependence using GPs)
- Optimise weights to agree with target "importance" using numerical algorithm

Back to the beginning

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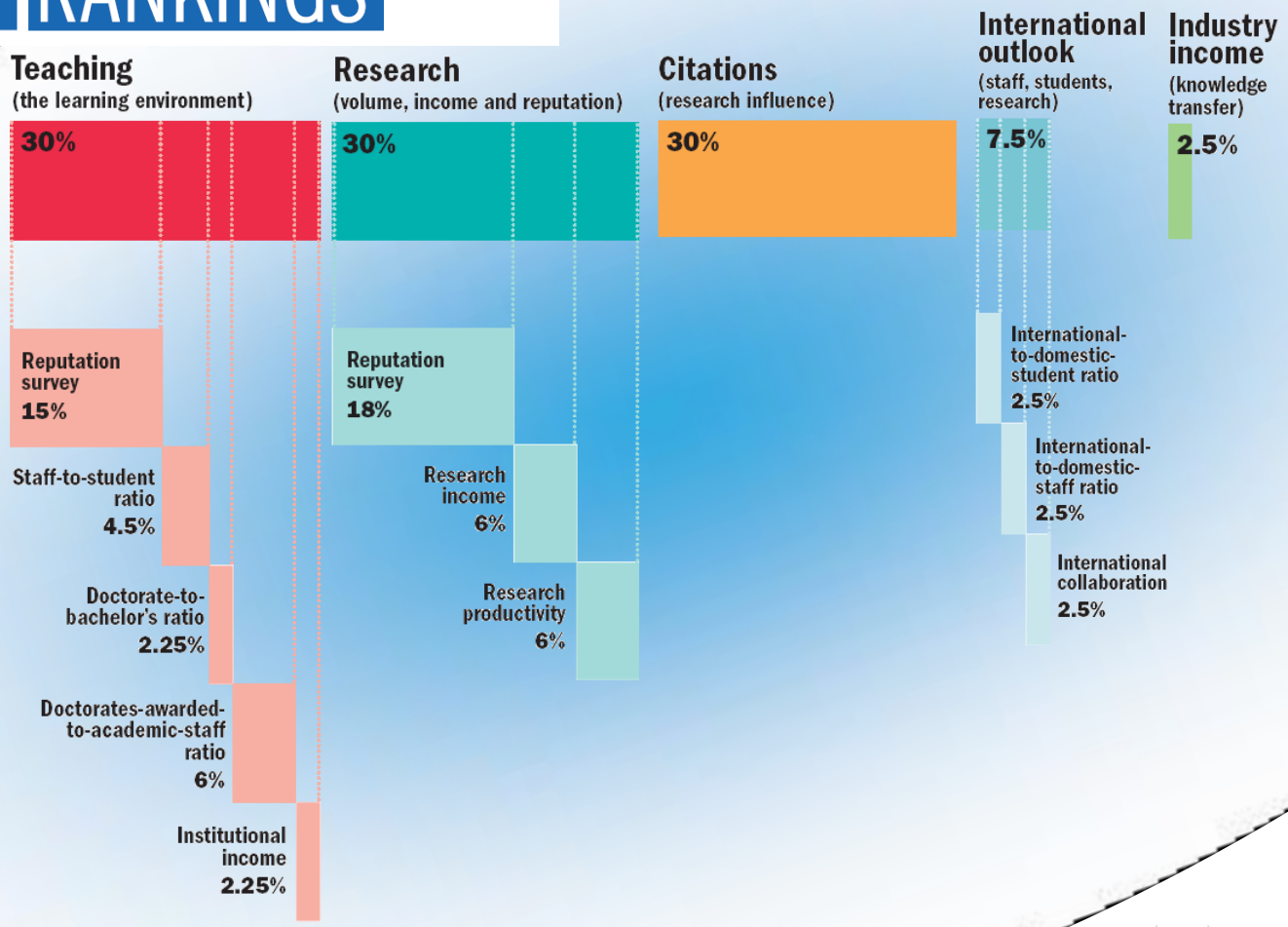
Academic Reputation	Employer Reputation	Faculty/ Student Ratio	Citations per Faculty	International Faculty	International Students
40%	10%	20%	20%	5%	5%
Opinion-based survey	Opinion-based survey	Teacher/ student ratio	Citations divided by faculty	Proportion of international faculty members	Proportion of international students

THE WORLD UNIVERSITY RANKINGS

TIMES HIGHER EDUCATION



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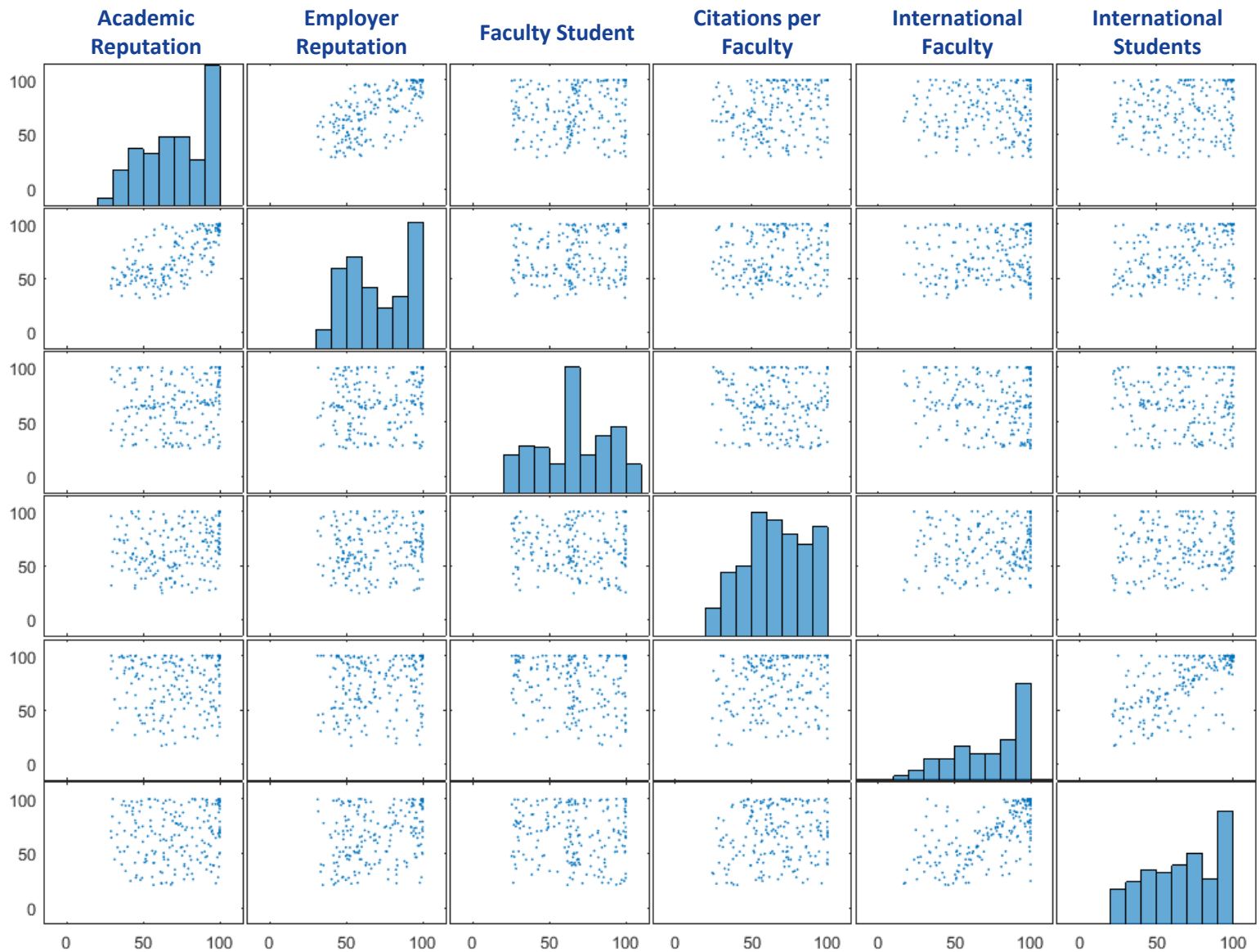
QS input data



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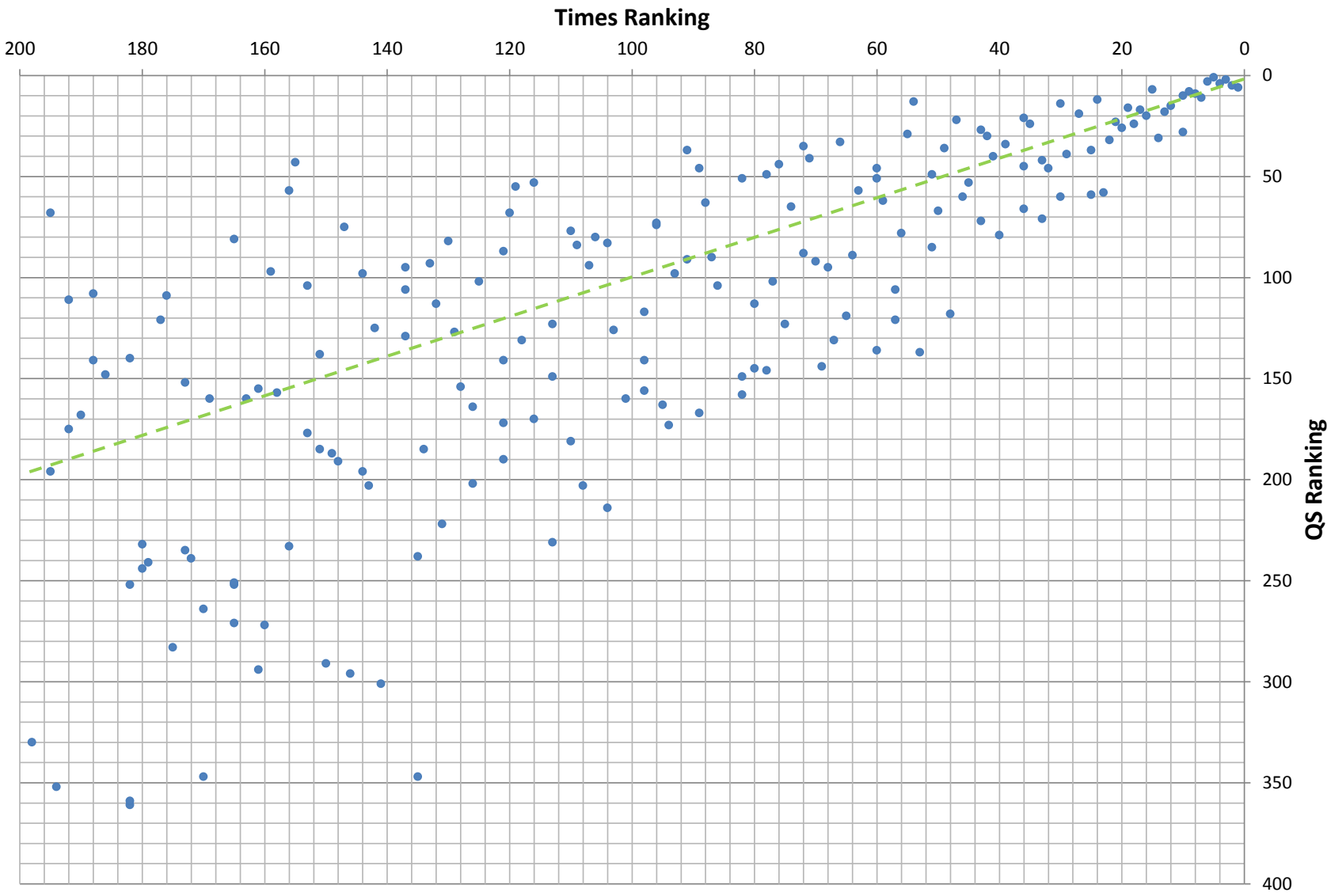
Times input data

European Commission



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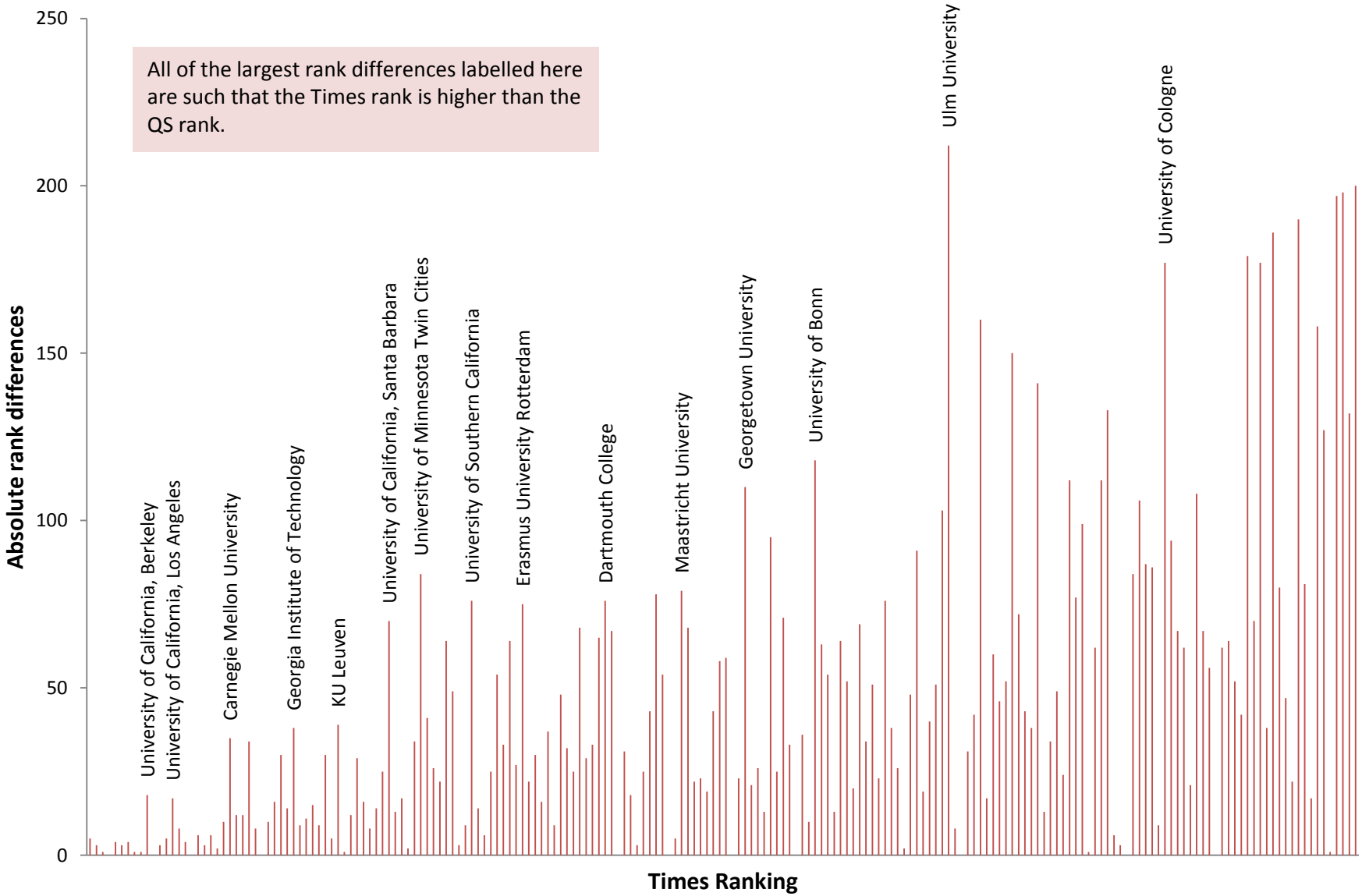

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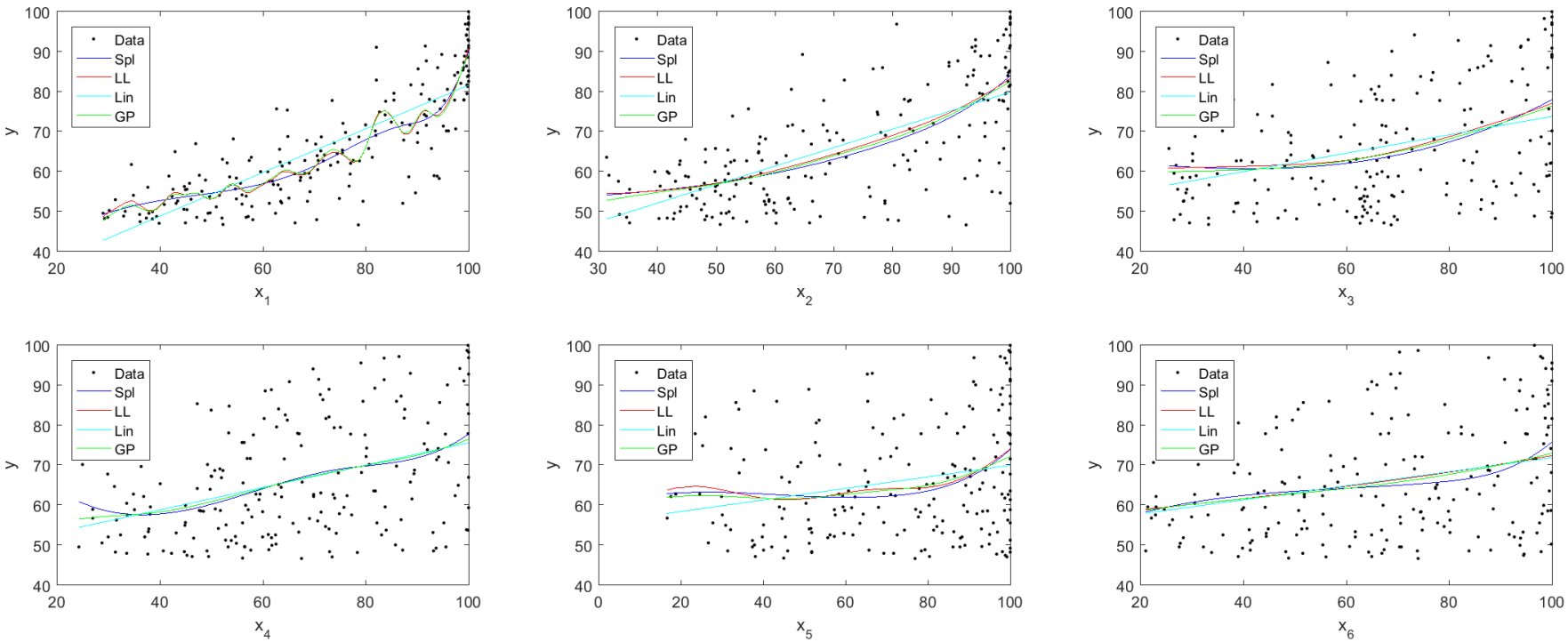
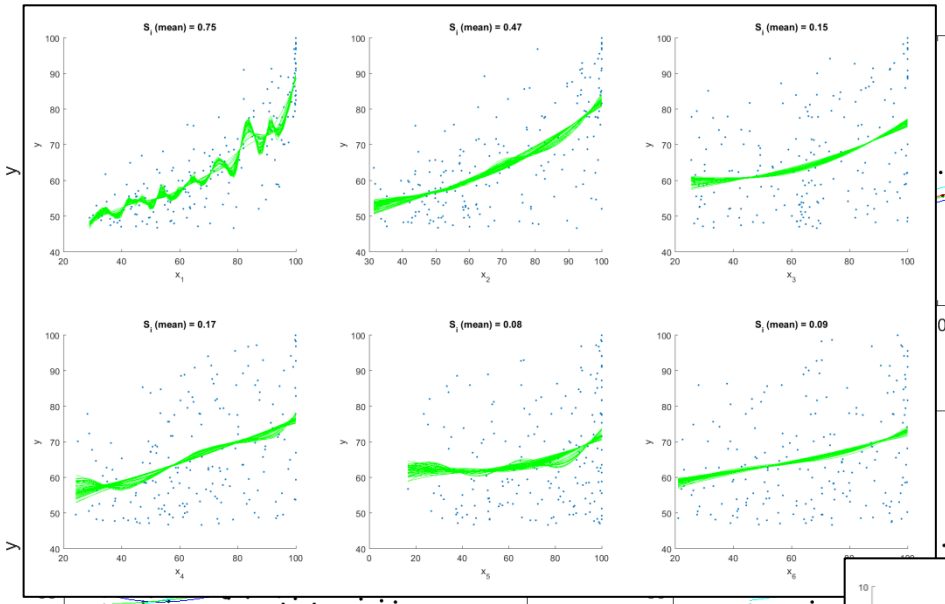
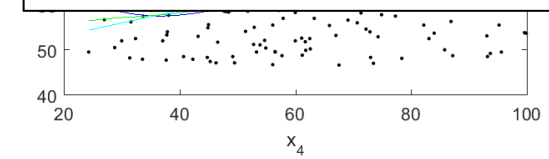
QS Regression

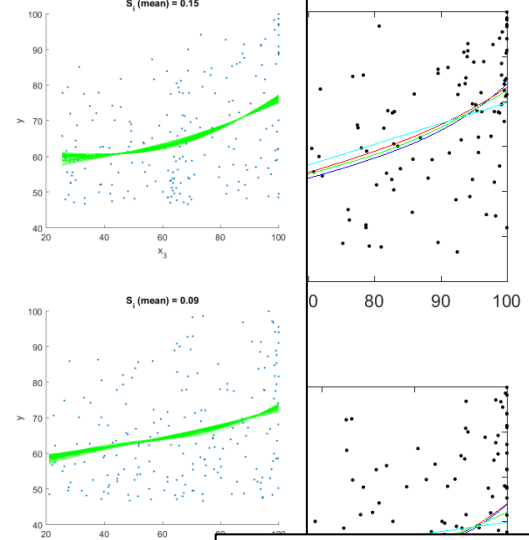


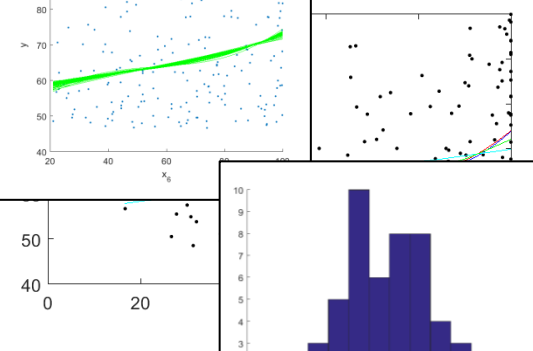
x1	Academic Reputation
x2	Employer Reputation
x3	Faculty Student
x4	Citations per Faculty
x5	International Faculty
x6	International Students

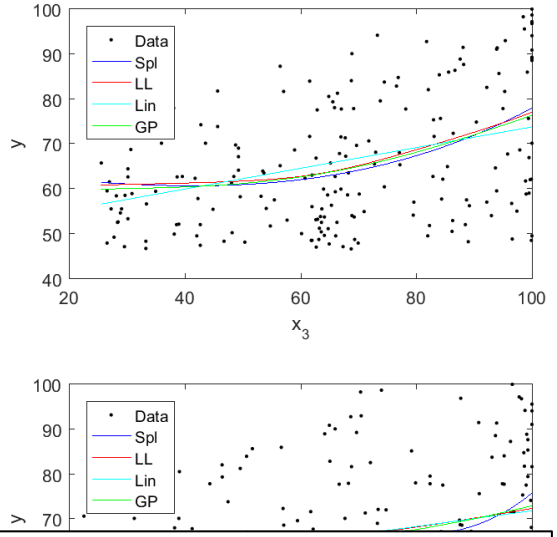
QS Regression

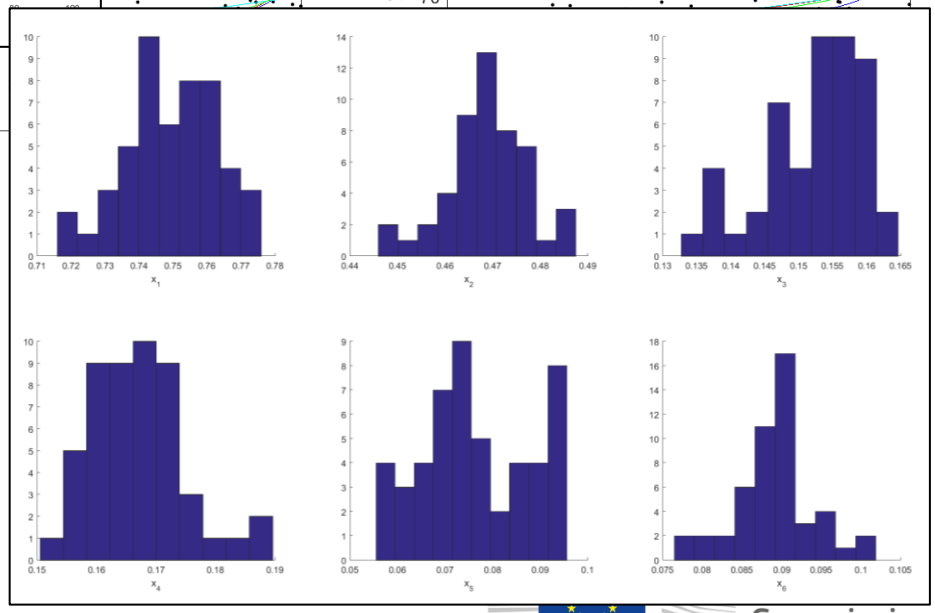












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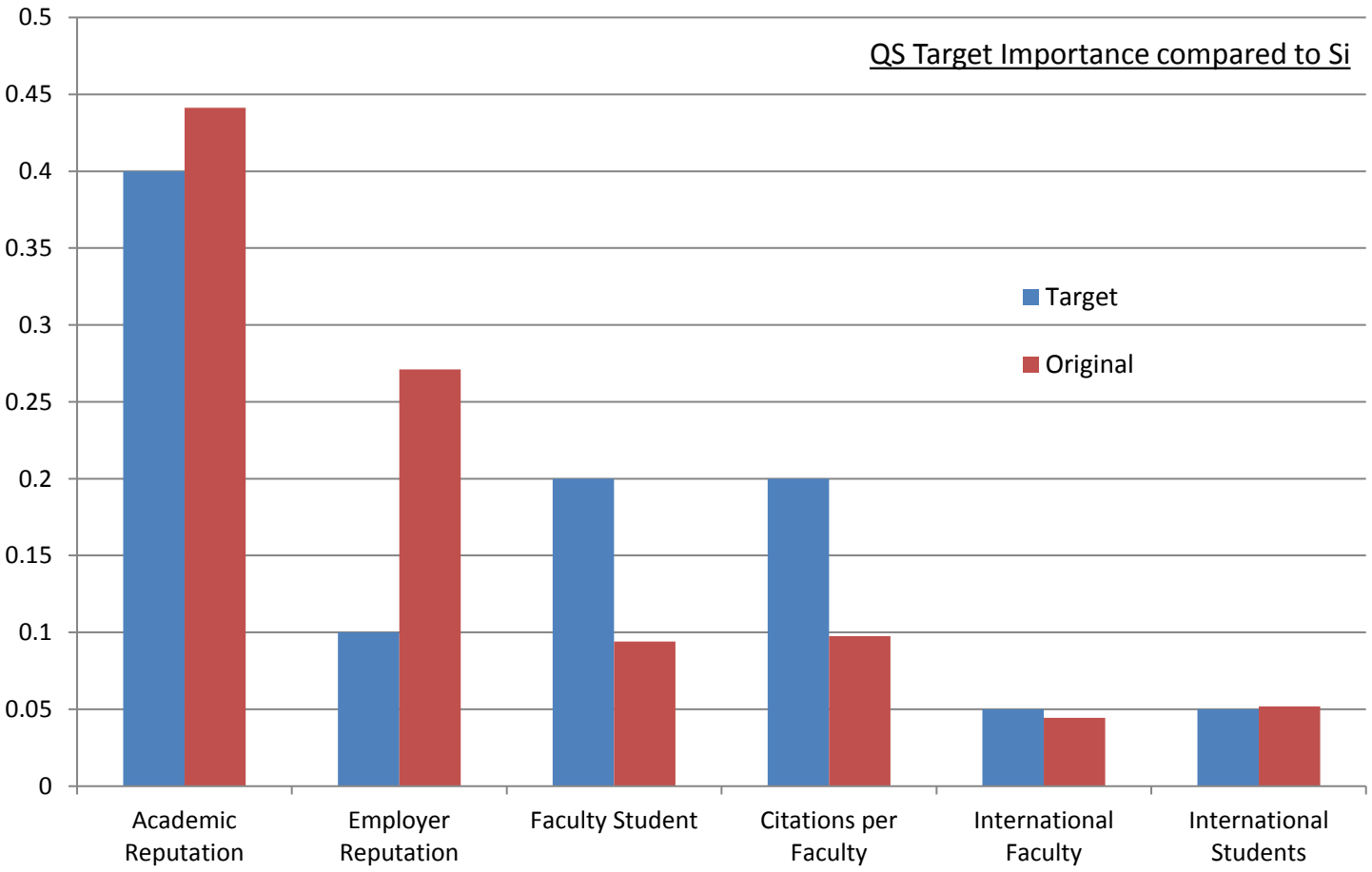

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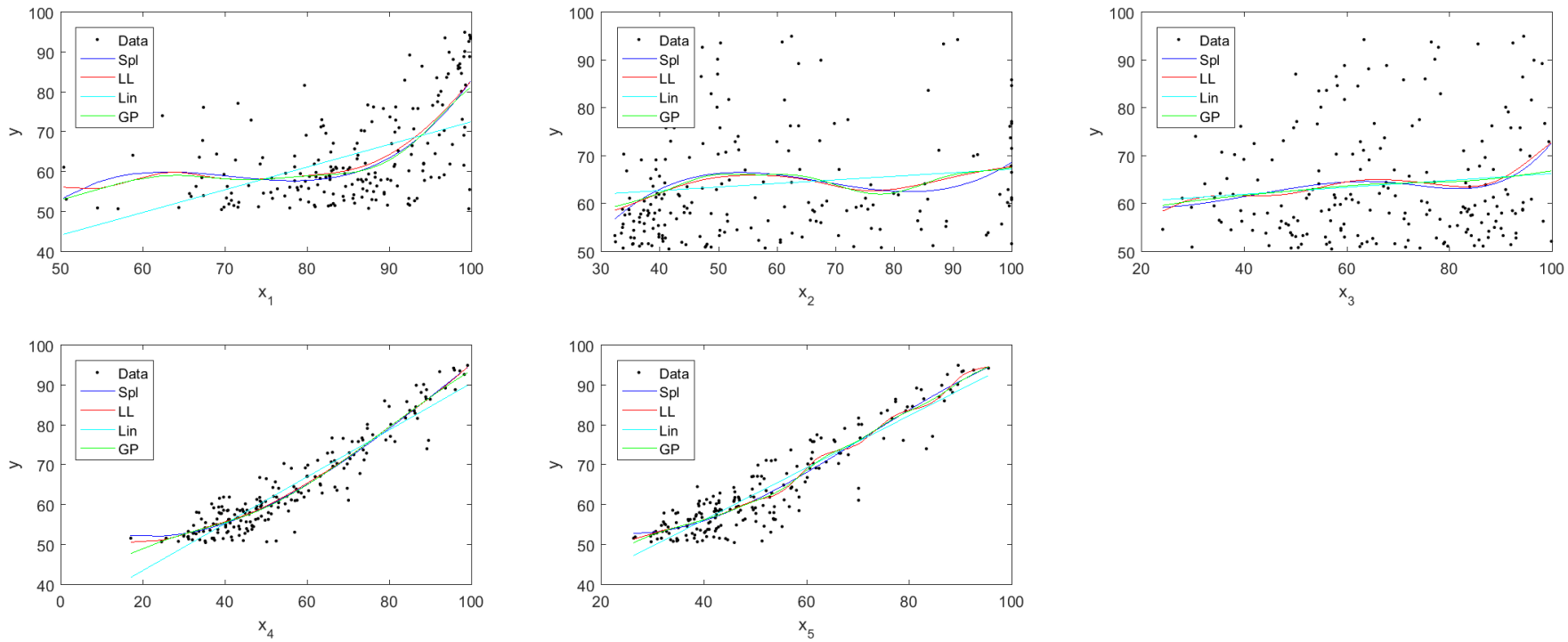
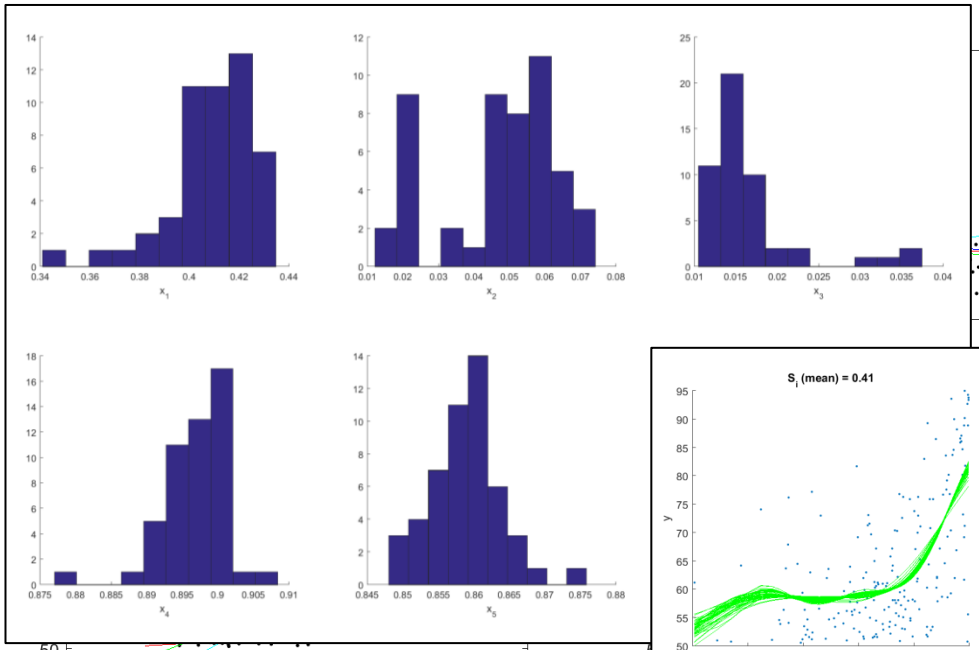
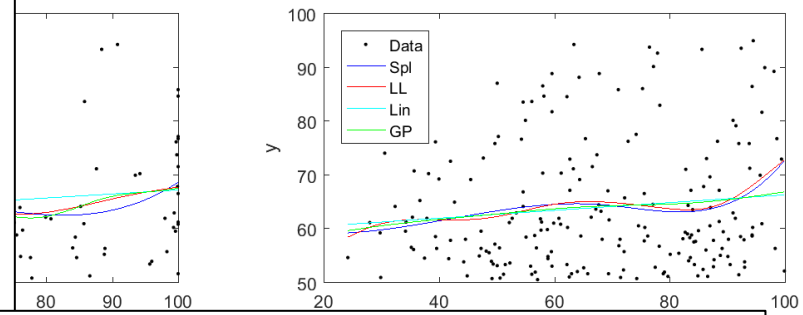
Times Regression

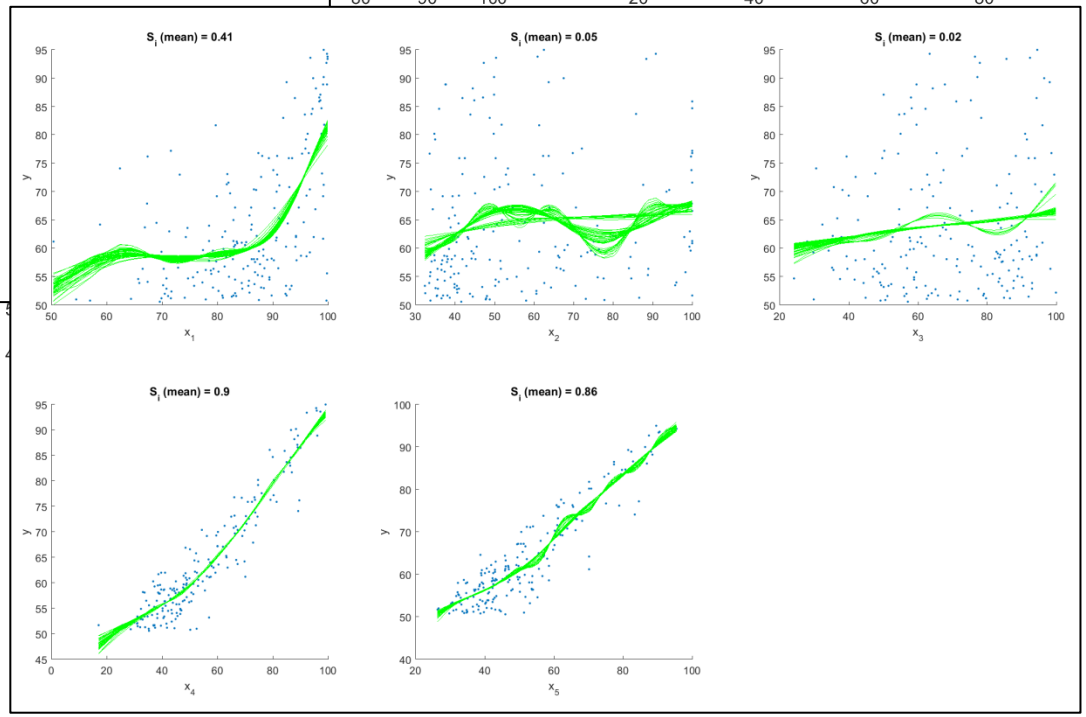


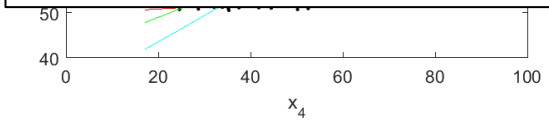
x1	Citations
x2	Industry Income
x3	International Outlook
x4	Research
x5	Teaching

Times Regression









x1	Citations
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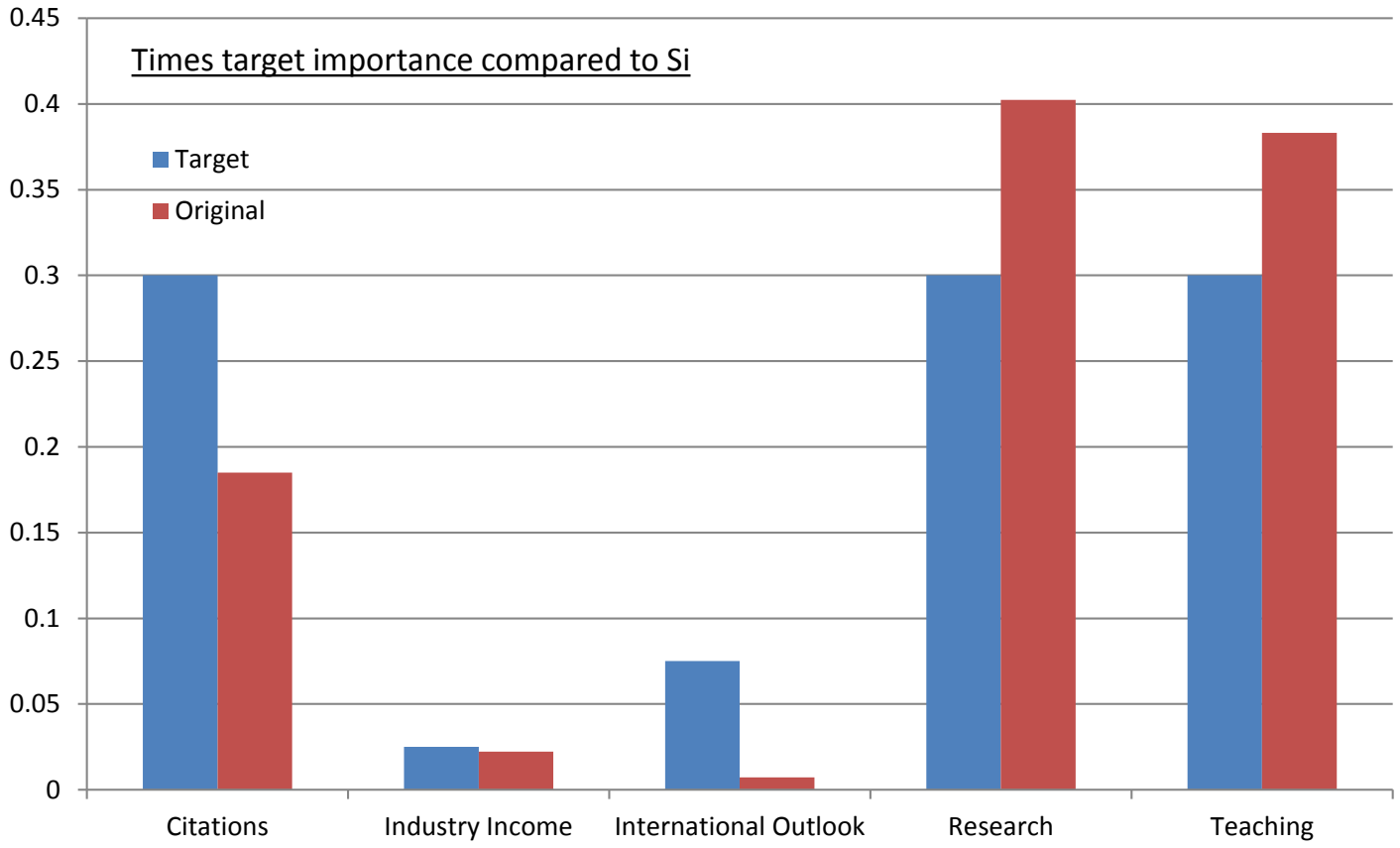
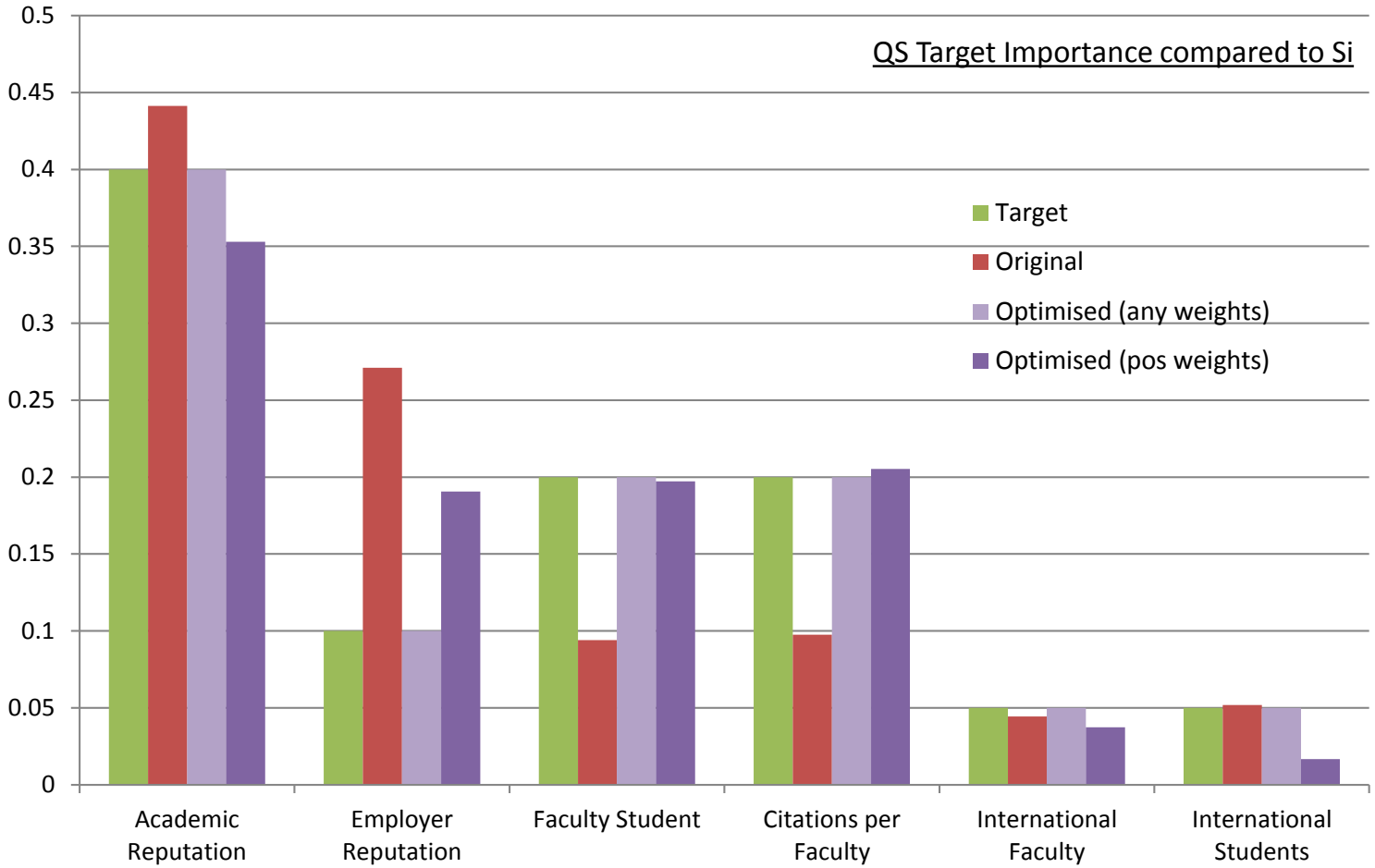
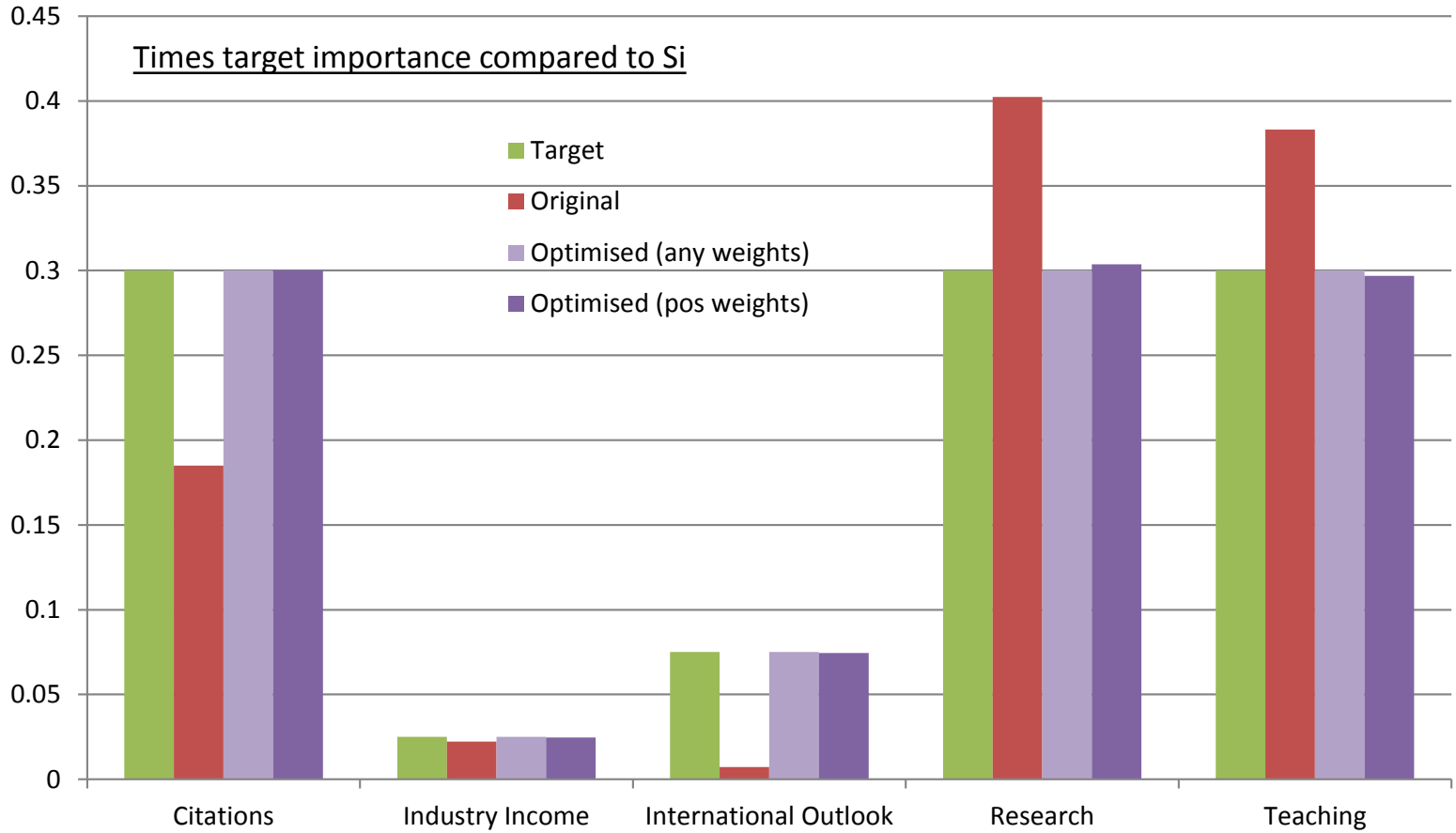

Times target importance compared to Si
Target
Original
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Citations
Industry Income
International Outlook
Research
Teaching



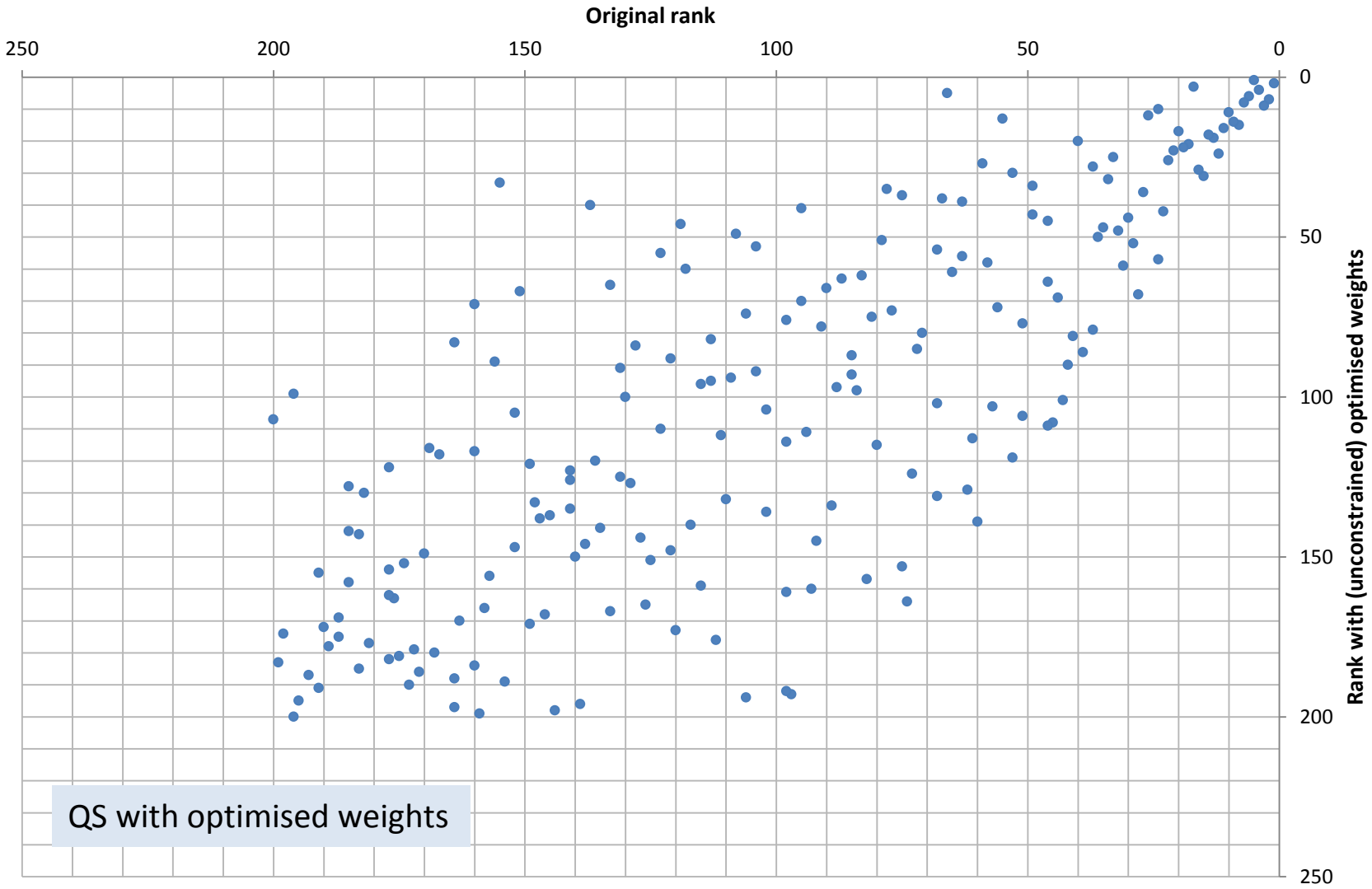
Original	0.4	0.1	0.2	0.2	0.05	0.05
Optimised	0.60	-0.37	0.32	0.28	-0.03	0.20
Opt (+)	0.32	0.00	0.30	0.31	0.07	0.00



Original	0.30	0.03	0.08	0.30	0.30
Optimised	0.46	-0.08	0.19	0.03	0.39
Opt (+)	0.44	0.11	0.16	0.00	0.29

			S_i				S_i,u	S_i,c			
			Linear	Spline	Loc. Lin.	GP	Spline	Linear	Spline	Loc. Lin.	GP
QS	x1	Academic Reputation	0.67	0.75	0.78	0.75	0.27	0.40	0.48	0.51	0.48
	x2	Employer Reputation	0.45	0.48	0.48	0.46	0.07	0.38	0.41	0.40	0.39
	x3	Faculty Student	0.14	0.17	0.15	0.16	0.12	0.02	0.05	0.03	0.04
	x4	Citations per Faculty	0.16	0.17	0.16	0.17	0.08	0.08	0.09	0.08	0.08
	x5	International Faculty	0.06	0.10	0.09	0.08	0.04	0.01	0.05	0.05	0.03
	x6	International Students	0.09	0.10	0.08	0.09	0.07	0.01	0.03	0.01	0.02
Times	x1	Citations	0.27	0.43	0.43	0.41	0.10	0.17	0.34	0.33	0.32
	x2	Industry Income	0.02	0.05	0.04	0.05	0.01	0.01	0.05	0.04	0.04
	x3	International Outlook	0.01	0.03	0.03	0.02	0.02	0.00	0.01	0.02	0.00
	x4	Research	0.88	0.90	0.90	0.90	0.04	0.84	0.86	0.86	0.86
	x5	Teaching	0.85	0.86	0.86	0.85	0.05	0.80	0.81	0.82	0.81

- NL regression estimates are fairly similar but linear regression not always sufficient
- Uncorrelated part can be dominated by correlated part: more likely that variable is not contributing as intended *and* difficult to optimise



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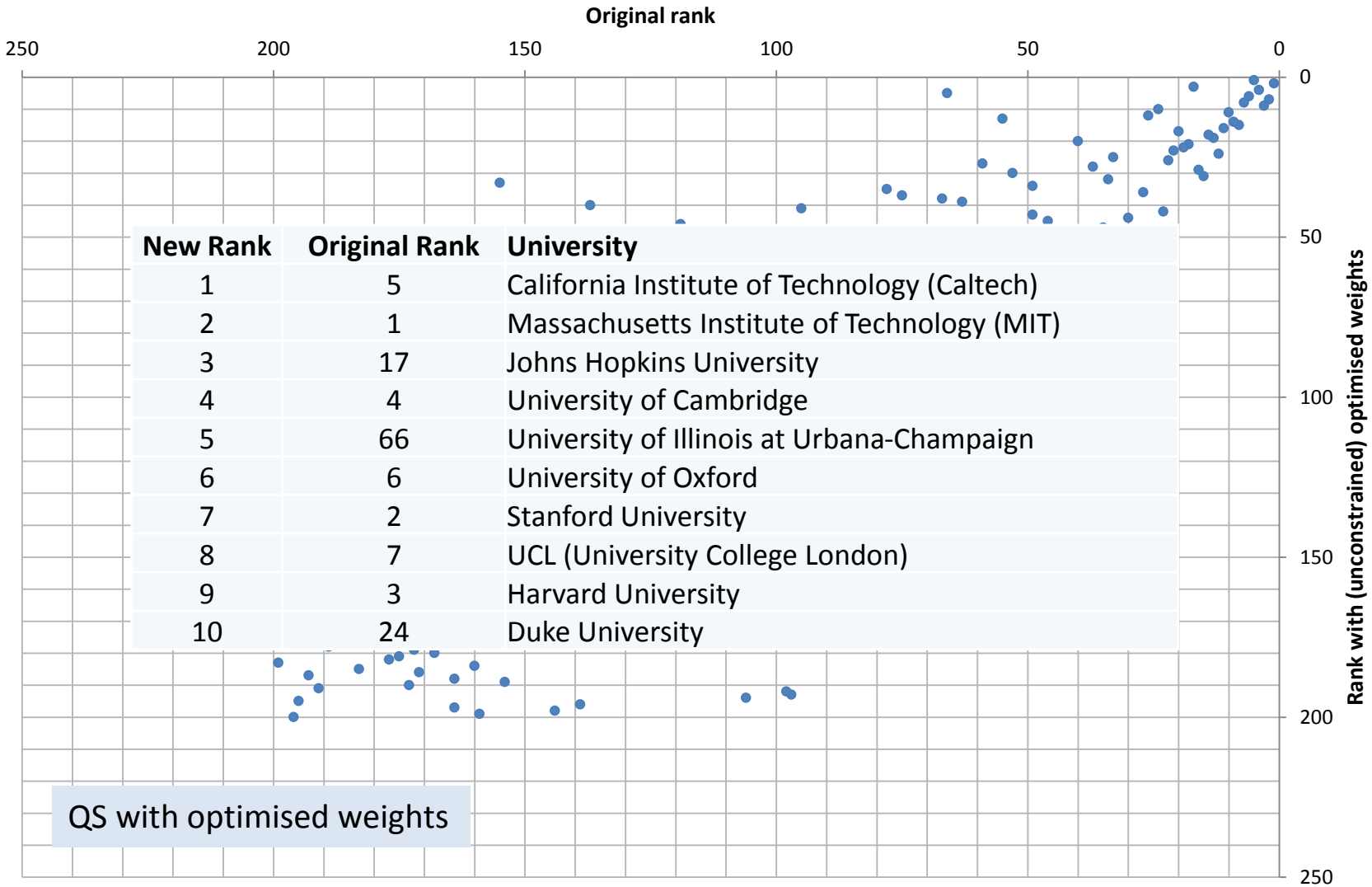
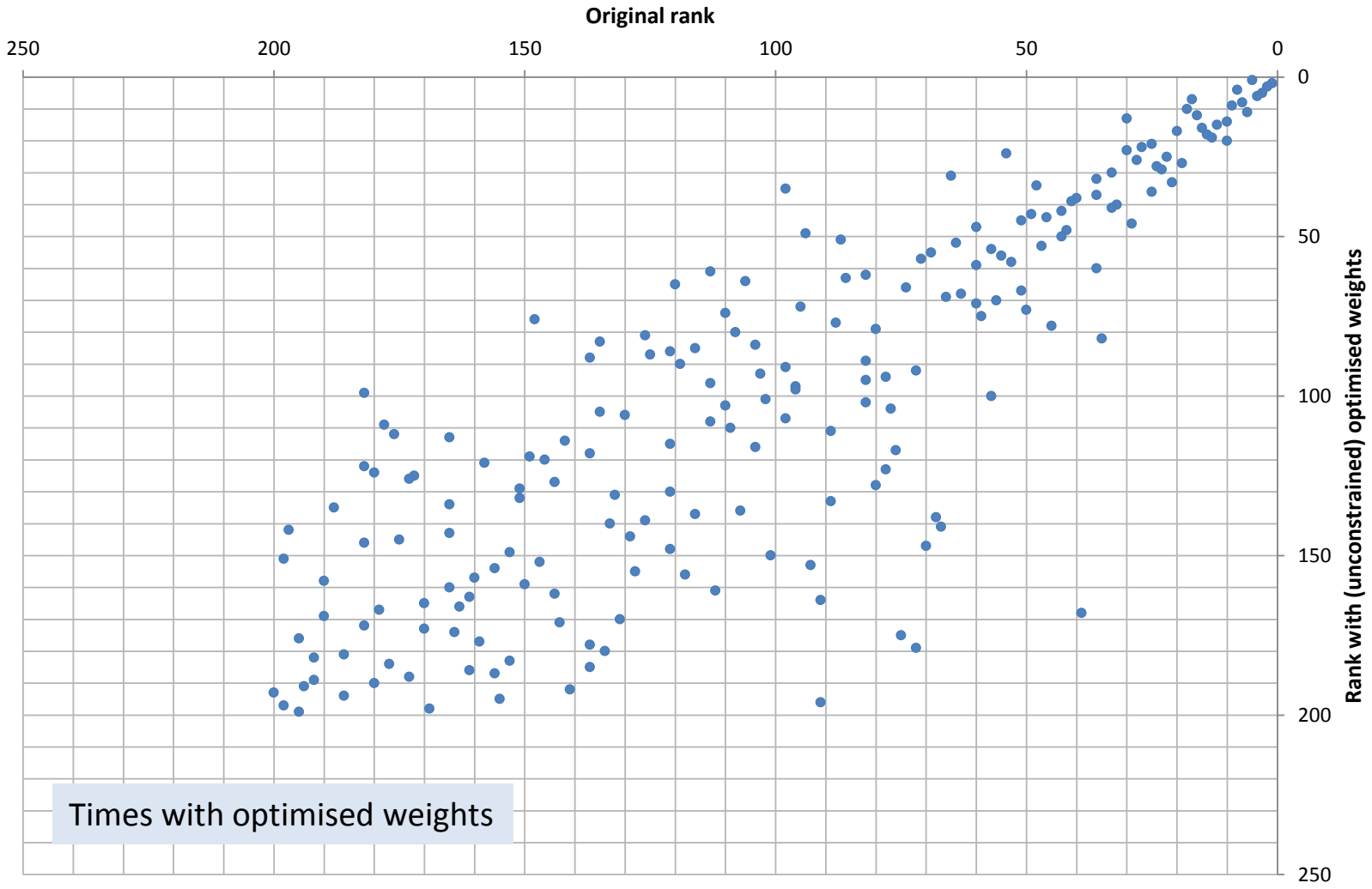


New Rank	Original Rank	University
1	5	California Institute of Technology (Caltech)
2	1	Massachusetts Institute of Technology (MIT)
3	17	Johns Hopkins University
4	4	University of Cambridge
5	66	University of Illinois at Urbana-Champaign
6	6	University of Oxford
7	2	Stanford University
8	7	UCL (University College London)
9	3	Harvard University
10	24	Duke University



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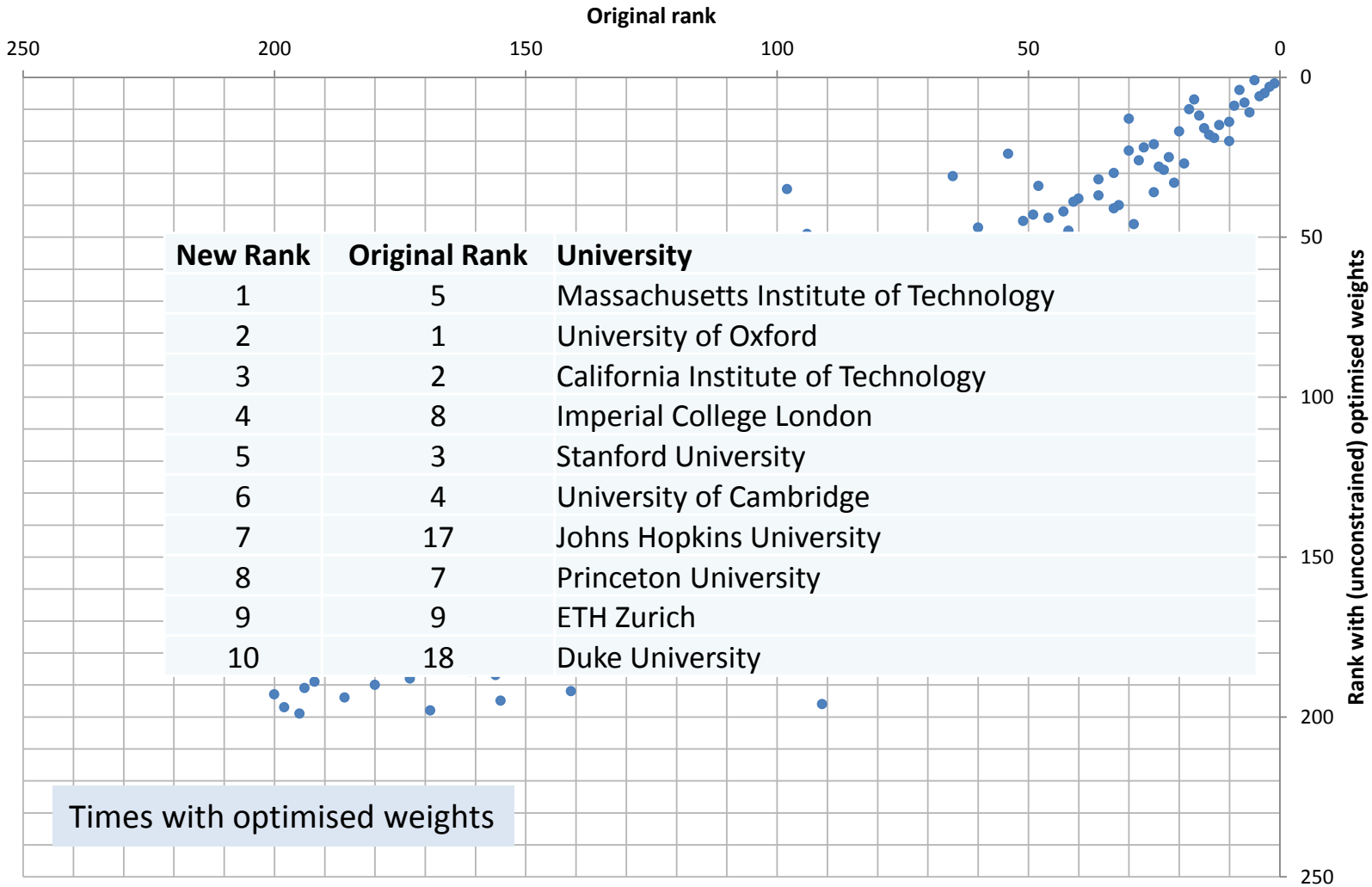

New Rank	Original Rank	University
1	5	Massachusetts Institute of Technology
2	1	University of Oxford
3	2	California Institute of Technology
4	8	Imperial College London
5	3	Stanford University
6	4	University of Cambridge
7	17	Johns Hopkins University
8	7	Princeton University
9	9	ETH Zurich
10	18	Duke University

Times with optimised weights

Conclusions

Weights don't equal importance in composite indicators.

Sensitivity analysis can help to explore and improve rankings

Rankings *very* sensitive to assumptions. Many other assumptions to explore here.

Open Questions

Does importance = S_i?

When we assign "importances" to variables, are we implicitly taking some correlation into account?

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